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A new estimate for the constants of an inequality due to Hardy and Littlewood

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ABSTRACT

In this paper we provide a family of inequalities, extending a recent result due to Albuquerque et al.

1. Introduction

The Hardy–Littlewood inequalities [11] for m-linear forms and polynomials (see [2–5, 7,9,13–15,17]) are perfect extensions of the Bohnenblust–Hille inequality [6] when the sequence space c_0 is replaced by the sequence space ℓ_p. These inequalities assert that for any integer $m \geq 2$ there exist constants $C^{\mathbb{K}}_{m,p}, D^{\mathbb{K}}_{m,p} \geq 1$ such that

$$\left(\sum_{j_1,\cdots,j_m=1}^{\infty} |T(e_{j_1},\cdots,e_{j_m})|^{\frac{2mp}{mp+p-2m}} \right)^{\frac{mp+p-2m}{2mp}} \leq C^{\mathbb{K}}_{m,p} \|T\|, \tag{1.1}$$

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when $2m \leq p \leq \infty$, and

$$\left(\sum_{j_1,\cdots,j_m=1}^{\infty} |T(e_{j_1},\cdots,e_{j_m})|^{\frac{p}{p-m}} \right)^{\frac{p-m}{p}} \leq D_{m,p}^{\mathbb{K}} \|T\|,$$

when $m < p \leq 2m$, for all continuous m-linear forms $T : \ell_p \times \cdots \times \ell_p \to \mathbb{K}$ (here, and henceforth, $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$). Both exponents are optimal, *i.e.*, cannot be smaller without paying the price of a dependence on n arising on the respective constants. Following usual convention in the field, c_0 is understood as the substitute of ℓ_∞ when the exponent p goes to infinity.

The investigation of the optimal constants of the Hardy–Littlewood inequalities is closely related to the fashionable, mysterious and puzzling investigation of the optimal Bohnenblust–Hille inequality constants (see, for instance [12,15] and the references therein).

In this note we extend the following result of [1, Theorem 3]:

Theorem 1 *(Albuquerque et al.). Let $m \geq 2$ be a positive integer and $m < p \leq 2m-2$. Then, for all continuous m-linear forms $T : \ell_p \times \cdots \times \ell_p \to \mathbb{K}$ and all positive integers n, we have*

$$\left(\sum_{j_i=1}^{n} \left(\sum_{\widehat{j_i}=1}^{n} |T(e_{j_1},\ldots,e_{j_m})|^{\frac{p}{p-(m-1)}} \right)^{\frac{p-(m-1)}{p}\cdot\frac{p}{p-m}} \right)^{\frac{p-m}{p}} \leq 2^{\frac{(m-1)(p-m+1)}{p}} \|T\|.$$

More precisely, using a variant of the technique introduced in [1], we find a family of inequalities extending the above result. Our result reads as follows, where A_{λ_0} is the optimal constant of the Khinchin inequality (defined in Section 2):

Theorem 2. *If $\lambda_0 \in [1,2)$ and*

$$\lambda_0 m < p \leq \frac{2\lambda_0(m-1)}{2-\lambda_0},$$

then

$$\left(\sum_{j_i=1}^{n} \left(\sum_{\widehat{j_i}=1}^{n} |T(e_{j_1},\ldots,e_{j_m})|^{s} \right)^{\frac{1}{s}\eta_1} \right)^{\frac{1}{\eta_1}} \leq A_{\lambda_0}^{\frac{-2(m-1)}{s}} \|T\|$$

for