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Random exponential attractor for cocycle and application to non-autonomous stochastic lattice systems with multiplicative white noise ☆

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Abstract

We first establish some sufficient conditions for constructing a random exponential attractor for a continuous cocycle on a separable Banach space and weighted spaces of infinite sequences. Then we apply our abstract result to study the existence of random exponential attractors for non-autonomous first order dissipative lattice dynamical systems with multiplicative white noise.

MSC: 37L55; 60H15; 35B40

Keywords: Random exponential attractor; Cocycle; Stochastic lattice system; Fractal dimension; Multiplicative white noise

1. Introduction

It is well known that the theory of attractors (including the global attractor, pullback attractor or kernel sections, uniform attractor, exponential attractor, pullback and uniform exponential attractor) for deterministic autonomous and non-autonomous dynamical systems or evolution

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equations has been developed intensively since late seventies of the last century, see [2,4,11,12,14,20,21,24,25,30,31,34,37,38,40–44,46].

When dealing with effects of uncertainty or noise from natural phenomena, the study of stochastic evolution equations has attracted lots of interests from both mathematicians and physicists [1,16,17]. The random attractor, was first studied by Ruelle [32], is an important concept to describe asymptotic behavior for a random dynamical system and to capture the essential dynamics with possibly extremely wide fluctuations. Since the mid-90s of the last century, there have been many publications concerning the theory of random attractors (mainly on existence, semicontinuity and bound of Hausdorff/fractal dimensions) and applications to stochastic evolution equations (such as Navier–Stokes equation, reaction–diffusion equations, wave equations and lattice systems with noise), see [3,5,6,8,9,16–19,23,26,27,29,33,36,39,47,48] and the references wherein.

However, there is an intrinsic drawback that random attractor sometimes attracts orbits at a relatively slow rate so that it takes an unexpected long time to reach it. Moreover, in general, it is usually difficult to estimate the attracting rate in terms of physical parameters of the studied system. And the attractor is possible sensitive to perturbations which makes it unobservable in experiments and numerical simulations. To overcome this drawback, Shirikyan and Zelik in [33] introduced the concept of random exponential attractor, which has finite fractal dimension and attracts exponentially any trajectory and is positively invariant, then it contains random attractor and become an appropriate alternative to study the asymptotic behavior of random dynamical systems. And [33] presents some sufficient conditions for constructing a random exponential attractor for an autonomous random dynamical system and application to nonlinear reaction–diffusion system with a random perturbation. But the method or conditions given in [33] is not easy to be verified for some stochastic partial differential equations and lattice systems driven by white noises.

We notice that the evolution mode of states in a random system is, in some sense, similar to the deterministic non-autonomous one and there were several construction methods to obtain a pullback exponential attractor for a (deterministic) process, see [11,12,20,21,25,30,43]. We also notice that there is a fundamental difference between random system and deterministic one. In contrast to the deterministic case, a trajectory of a random system is often unbounded in time (explicitly, along the path of sample point). Thus, if no imposition some "strongly" restriction on the system, then the constants in appropriate squeezing property (playing a key role in the construction of an exponential attractor) will depend on time (hence, be unbounded), and so, a trivial straightforward extension from deterministic system to random system does not work. Fortunately, some time averages of these quantities can be bounded and possibly controlled, which provides a useful way for constructing an exponential attractor for a random system.

In this article, motivated by ideas of [25,33,43,47], we first establish some sufficient conditions for the existence and construction of a random exponential attractor for a continuous cocycle on a separable Banach space and weighted spaces of infinite sequences. Here it is worth mentioning that our conditions just need to check the boundedness of some random variables in the mean and can be easily verified for some stochastic evolution equations.

Recently, lattice dynamical systems (LDSs) (or ordinary differential equations on infinite lattices) have drawn much attention from researchers because of their wide range of applications in various areas (e.g. [13,15]). Since Bates et al. [4] in 2001 presented a framework on the existence and upper semicontinuity of a global attractor associated with autonomous first-order LDSs, there have been a lot of publications concerning various attractors (including global attractor, uniform attractor, pullback attractor or kernel section, exponential attractor, pullback and uniform ex-