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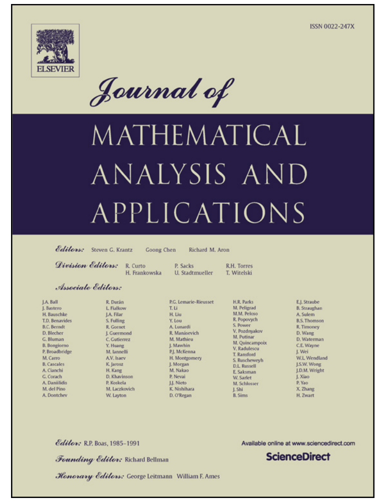

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COMBINING FAST INERTIAL DYNAMICS FOR CONVEX OPTIMIZATION WITH TIKHONOV REGULARIZATION.

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Abstract

In a Hilbert space setting $\mathcal{H}$, we study the convergence properties as $t \to +\infty$ of the trajectories of the second-order differential equation

$$\text{(AVD)}_{\alpha,\epsilon} \qquad \ddot{x}(t) + \frac{\alpha}{t}\dot{x}(t) + \nabla\Phi(x(t)) + \epsilon(t)x(t) = 0,$$

where $\nabla\Phi$ is the gradient of a convex continuously differentiable function $\Phi : \mathcal{H} \to \mathbb{R}$, α is a positive parameter, and $\epsilon(t)x(t)$ is a Tikhonov regularization term, with $\epsilon(t)$ positive, and $\lim_{t\to\infty} \epsilon(t) = 0$. In this damped inertial system, the damping coefficient $\frac{\alpha}{t}$ vanishes asymptotically, but not too quickly, a key property to obtain rapid convergence of the values. In the case $\epsilon(\cdot) \equiv 0$, this dynamic has been highlighted recently by Su, Boyd, and Candès as a continuous version of the Nesterov accelerated gradient method. Depending on the speed of convergence of $\epsilon(t)$ to zero, we analyze the convergence properties of the trajectories of $\text{(AVD)}_{\alpha,\epsilon}$. We obtain results ranging from the rapid convergence of $\Phi(x(t))$ to $\min\Phi$ when $\epsilon(t)$ decreases rapidly to zero, up to the strong convergence of the trajectories to the element of minimum norm of the set of minimizers of Φ, when $\epsilon(t)$ tends slowly to zero. When $\epsilon(t) = \frac{1}{t^r}$, the critical value of r separating the two above cases is $r = 2$.

Keywords: Convex optimization; hierarchical minimization; inertial dynamics; Nesterov accelerated gradient method; Tikhonov approximation; vanishing viscosity.

1. Introduction

Throughout the paper, $\mathcal{H}$ is a real Hilbert space which is endowed with the scalar product $\langle \cdot, \cdot \rangle$, with $\|x\|^2 = \langle x, x \rangle$ for $x \in \mathcal{H}$. Let $\Phi : \mathcal{H} \to \mathbb{R}$ be a convex differentiable function. We consider the convex minimization problem

$$\min\left\{\Phi(x) : \ x \in \mathcal{H}\right\}, \tag{1}$$

whose solution set $S = \operatorname{argmin}\Phi$ is supposed to be nonempty. We aim at finding by rapid methods the element of minimum norm of the closed convex set S. To that end, we study the asymptotic behaviour (as $t \to +\infty$) of the trajectories of the second-order differential equation

$$\text{(AVD)}_{\alpha,\epsilon} \qquad \ddot{x}(t) + \frac{\alpha}{t}\dot{x}(t) + \nabla\Phi(x(t)) + \epsilon(t)x(t) = 0, \tag{2}$$

where α is a positive parameter, and $\epsilon(t)x(t)$ is a Tikhonov regularization term. Throughout the paper (unless otherwise stated), we assume that

(H_1) $\Phi : \mathcal{H} \to \mathbb{R}$ is convex and differentiable, its gradient $\nabla\Phi$ is Lipschitz continuous on bounded sets.

(H_2) $S := \operatorname{argmin}\Phi \neq \emptyset$.

(H_3) $\epsilon : [t_0, +\infty[\to \mathbb{R}^+$ is a nonincreasing function, of class $\mathcal{C}^1$, such that $\lim_{t\to\infty} \epsilon(t) = 0$.