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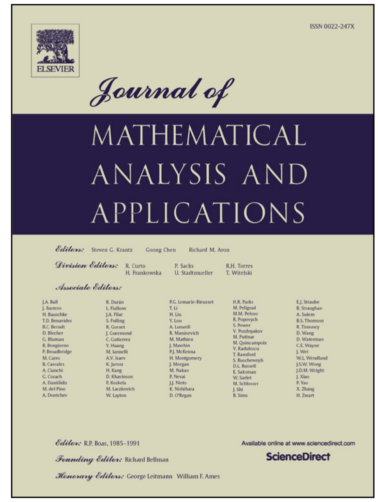

Value function, relaxation, and transversality conditions in infinite horizon optimal control

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VALUE FUNCTION, RELAXATION, AND TRANSVERSALITY CONDITIONS IN INFINITE HORIZON OPTIMAL CONTROL*

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ABSTRACT. We investigate the value function $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+ \cup \{+\infty\}$ of the infinite horizon problem in optimal control for a general—not necessarily discounted—running cost and provide sufficient conditions for its lower semicontinuity, continuity, and local Lipschitz regularity. Then we use the continuity of $V(t, \cdot)$ to prove a relaxation theorem and to write the first order necessary optimality conditions in the form of a, possibly abnormal, maximum principle whose transversality condition uses limiting/horizontal supergradients of $V(0, \cdot)$ at the initial point. When $V(0, \cdot)$ is merely lower semicontinuous, then for a dense subset of initial conditions we obtain a normal maximum principle augmented by sensitivity relations involving the Fréchet subdifferentials of $V(t, \cdot)$. Finally, when V is locally Lipschitz, we prove a normal maximum principle together with sensitivity relations involving generalized gradients of V for arbitrary initial conditions. Such relations simplify drastically the investigation of the limiting behaviour at infinity of the adjoint state.

Keywords. Infinite horizon problem, value function, relaxation theorem, sensitivity relation, maximum principle.

1. INTRODUCTION

In some models of mathematical economics one encounters the following infinite horizon optimal control problem

$$W(x_0) = \inf \int_0^\infty e^{-\lambda t} \ell(x(t), u(t))\, dt$$

over all trajectory-control pairs (x, u), subject to the state equation

$$\begin{cases} x'(t) = f(x(t), u(t)), \quad u(t) \in U \quad \text{for a.e. } t \geq 0 \\ x(0) = x_0, \end{cases}$$

where controls $u(\cdot)$ are Lebesgue measurable functions and $\lambda > 0$. (Usually such models involve "sup" instead of "inf." However, redefining the cost function, we can always replace a maximization problem by a minimization one.) Its history goes back to Ramsey [18]. The term $e^{-\lambda t}$ is sometimes called a discount factor. The literature addressing this problem deals with traditional questions of existence of optimal solutions, regularity of W, necessary and sufficient optimality conditions. Usually assumptions are imposed to ensure the local Lipschitz continuity of W.

The question of necessary conditions is quite challenging, because unlike for classical finite horizon problems, transversality conditions are not immediate. Indeed, let $(\bar{x}, \bar{u})$ be a given optimal trajectory-control pair. It is well known that if ∞ in the above problem is replaced by some $T > t_0$, that is, the infinite horizon problem is reduced to the Bolza one

$$\text{minimize} \int_0^T e^{-\lambda t} \ell(x(t), u(t))\, dt \tag{1.1}$$

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