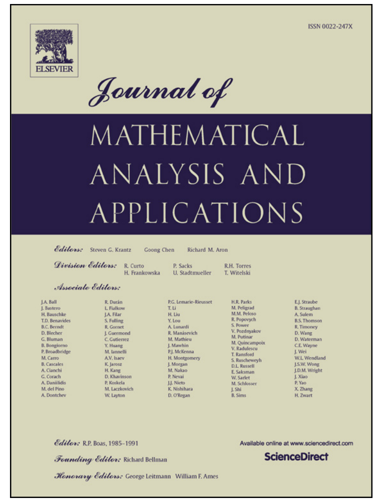

Interior Controllability of Semilinear Degenerate Wave Equations

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Interior Controllability of Semilinear Degenerate Wave Equations*

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Abstract

In this paper, we prove the existence of interior controls for one-dimensional semilinear degenerate wave equations. By duality argument, we reduce the problem to an observability estimate for the linear degenerate wave equation. First, the unique continuation for the degenerate wave equation is established. By means of this and the multiplier method, we obtain the observability estimate.

Key Words. Controllability, observability, unique continuation, degenerate wave equation

1 Introduction and main result

Let us consider the following semilinear degenerate wave equation:

$$\begin{cases} y_{tt} - (x^p y_x)_x + f(y) = \chi_\omega h & (x,t) \in Q, \\ y(0,t) = 0, \ y(s,t) = 0 & t \in (0,T), \\ y(x,0) = y_0(x), \ y_t(x,0) = y_1(x) & x \in \Omega, \end{cases} \tag{1.1}$$

with $h \in L^2(\omega \times (0,T))$ denoting the control, y denoting the state, (y_0, y_1) being an arbitrary initial value, and $f \in C^1(\mathbb{R})$ being a globally Lispchitz continuous function. In (1.1), $T > 0$, $s > 0$, $\Omega = (0,s)$ and $Q = \Omega \times (0,T)$. Let $0 < \alpha < \beta < s$ and $\omega = (\alpha, \beta)$ be a nonempty open subset of Ω. χ_ω denotes the characteristic function of ω. Take $p \in (0,1)$.

To begin with, we define a linear space $H_p^1(\Omega)$ by:

$$H_p^1(\Omega) = \big\{ f \in L^2(\Omega) \ \big| \ f \text{ is absolutely continuous in } \overline{\Omega}, \ x^{\frac{p}{2}} f_x \in L^2(\Omega) \text{ and } f(0) = f(s) = 0 \big\}.$$

Then $H_p^1(\Omega)$ is a Hilbert space, whose inner product is

$$(f,g)_{H_p^1(\Omega)} = \int_\Omega (fg + x^p f_x g_x) dx, \quad \forall \ f, g \in H_p^1(\Omega).$$

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