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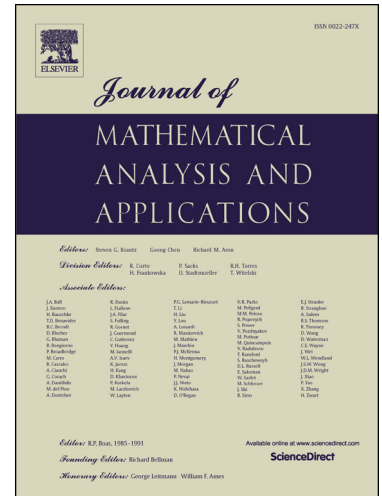
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APPROXIMATION OF MAXIMUM OF GAUSSIAN RANDOM FIELDS

ENKELEJD HASHORVA, OLEG SELEZNJEV, AND ZHONGQUAN TAN

Abstract: This contribution is concerned with Gumbel limiting results for supremum $M_n = \sup_{t \in [0, T_n]} |X_n(t)|$ with $X_n, n \in \mathbb{N}^2$ centered Gaussian random fields with continuous trajectories. We show first the convergence of a related point process to a Poisson point process thereby extending previous results obtained in [1] for Gaussian processes. Furthermore, we derive Gumbel limit results for M_n as $n \rightarrow \infty$ and show a second-order approximation for $\mathbb{E}\{M_n^p\}^{1/p}$ for any $p \geq 1$.

Key Words: Maxima; homogeneous Gaussian random fields; non-homogeneous Gaussian random fields; Pickands constant; Piterbarg constant; Gumbel limit theorem; Berman's inequality.

AMS Classification: Primary 60F05; secondary 60G15

1. INTRODUCTION

Classical results on extremes of Gaussian processes show that for $X(t), t \geq 0$ a centered Gaussian process with continuous sample paths, the tail asymptotic behaviour of $M(T) = \sup_{t \in [0, T]} X(t)$ can be exactly derived for some classes of both stationary and non-stationary X . Namely, if X is stationary with correlation function satisfying

$$(1) \quad r(t) = 1 - C|t|^\alpha + o(|t|^\alpha) \quad \text{as } t \rightarrow 0$$

for some $\alpha \in (0, 2]$, $C > 0$, and $r(t) < 1, \forall t \neq 0$, then (see [2, 3, 4])

$$(2) \quad \mathbb{P}(M(T) > u) \sim TC^{1/\alpha} \mathcal{H}_\alpha u^{2/\alpha} \mathbb{P}(X(0) > u) \quad \text{as } u \rightarrow \infty.$$

Here \sim means asymptotic equivalence of two real-valued functions when the argument tends to some specific point and \mathcal{H}_α is the Pickands constant defined by (see, e.g., [5, 4] and the recent contributions [6, 7])

$$\mathcal{H}_\alpha = \lim_{T \rightarrow \infty} T^{-1} \mathbb{E} \left\{ \sup_{t \in [0, T]} e^{\sqrt{2}B_\alpha(t) - t^\alpha} \right\} = \mathbb{E} \left\{ \frac{\sup_{t \in \mathbb{R}} e^{\sqrt{2}B_\alpha(t) - |t|^\alpha}}{\int_{t \in \mathbb{R}} e^{\sqrt{2}B_\alpha(t) - |t|^\alpha} dt} \right\},$$

with $\{B_\alpha(t), t \in \mathbb{R}\}$ a standard fractional Brownian motion (fBm) with Hurst index $\alpha/2$, i.e., B_α is a centered self-similar Gaussian process with Hurst index $\alpha/2$, stationary increments and $\mathbb{E}\{B_\alpha^2(t)\} = |t|^\alpha, t \in \mathbb{R}$.

If X is a centered non-stationary Gaussian process with continuous sample paths and variance function $\sigma^2(t)$ that has a unique point of maximum in $[0, T]$, say, 0 with $\sigma(0) = 1$, then in view of [4], we have

$$(3) \quad \mathbb{P}(M(T) > u) \sim K_{\alpha, \beta} u^{\max(0, 2/\alpha - 2/\beta)} \mathbb{P}(X(0) > u),$$

provided that for $\alpha \in (0, 2], a > 0$, the correlation function $r(s, t)$ satisfies

$$1 - r(s, t) = a|t - s|^\alpha + o(|t - s|^\alpha) \quad \text{as } s, t \rightarrow 0,$$

and for b, β positive

$$(4) \quad 1 - \sigma(t) = b|t|^\beta + o(|t|^\beta) \quad \text{as } t \rightarrow 0,$$

assuming additionally that X satisfies a global Hölder condition. Here $K_{\alpha, \beta} = 2\mathcal{H}_\alpha a^{1/\alpha} \Gamma(1/\beta + 1) b^{-1/\beta}$ with $\Gamma(\cdot)$ the Euler Gamma function, for $\alpha < \beta$; $K_{\alpha, \beta} = 1$ for $\alpha > \beta$ and $K_{\alpha, \alpha}$ equals the Piterbarg constant

$$\mathcal{H}_\alpha^{b/a} = \lim_{T \rightarrow \infty} \mathbb{E} \left\{ \sup_{t \in [0, T]} e^{\sqrt{2}B_\alpha(t) - (1+b/a)t^\alpha} \right\} \in (0, \infty).$$

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