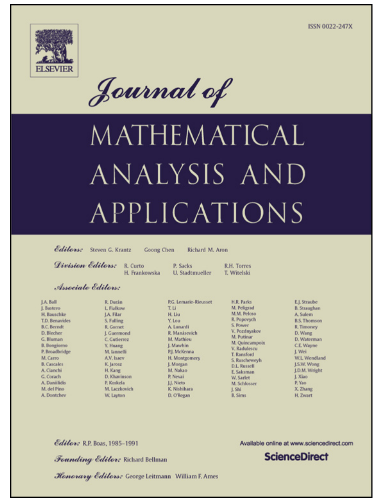
Accepted Manuscript



Approximation of Maximum of Gaussian Random Fields

Enkelejd Hashorva, Oleg Seleznjev, Zhongquan Tan

PII: S0022-247X(17)30797-7
DOI: http://dx.doi.org/10.1016/j.jmaa.2017.08.040
Reference: YJMAA 21647

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 23 December 2016

APPROXIMATION OF MAXIMUM OF GAUSSIAN RANDOM FIELDS

ENKELEJD HASHORVA, OLEG SELEZNJEV, AND ZHONGQUAN TAN

Abstract: This contribution is concerned with Gumbel limiting results for supremum $\mathbb{M}_{\boldsymbol{n}} = \sup_{\boldsymbol{t}\in[\boldsymbol{0},\boldsymbol{T_n}]}|X_{\boldsymbol{n}}(\boldsymbol{t})|$ with $X_{\boldsymbol{n}}, \boldsymbol{n} \in \mathbb{N}^2$ centered Gaussian random fields with continuous trajectories. We show first the convergence of a related point process to a Poisson point process thereby extending previous results obtained in [1] for Gaussian processes. Furthermore, we derive Gumbel limit results for $\mathbb{M}_{\boldsymbol{n}}$ as $\boldsymbol{n} \to \infty$ and show a second-order approximation for $\mathbb{E}\{\mathbb{M}_{\boldsymbol{n}}^p\}^{1/p}$ for any $p \ge 1$.

Key Words: Maxima; homogeneous Gaussian random fields; non-homogeneous Gaussian random fields; Pickands constant; Piterbarg constant; Gumbel limit theorem; Berman's inequality.
AMS Classification: Primary 60F05; secondary 60G15

1. Introduction

Classical results on extremes of Gaussian processes show that for $X(t), t \ge 0$ a centered Gaussian process with continuous sample paths, the tail asymptotic behaviour of $M(T) = \sup_{t\in[0,T]} X(t)$ can be exactly derived for some classes of both stationary and non-stationary X. Namely, if X is stationary with correlation function satisfying

$$r(t) = 1 - C|t|^\alpha + \mathrm{o}(|t|^\alpha) \quad \text{as } t \to 0 \tag{1}$$

for some $\alpha \in (0, 2]$, $C > 0$, and $r(t) < 1, \forall t \ne 0$, then (see [2, 3, 4])

$$\mathbb{P}(M(T) > u) \sim TC^{1/\alpha}\mathcal{H}_\alpha u^{2/\alpha}\mathbb{P}(X(0) > u) \quad \text{as } u \to \infty. \tag{2}$$

Here $\sim$ means asymptotic equivalence of two real-valued functions when the argument tends to some specific point and $\mathcal{H}_\alpha$ is the Pickands constant defined by (see, e.g., [5, 4] and the recent contributions [6, 7])

$$\mathcal{H}_\alpha = \lim_{T\to\infty} T^{-1}\mathbb{E}\left\{\sup_{t\in[0,T]} e^{\sqrt{2}B_\alpha(t)-t^\alpha}\right\} = \mathbb{E}\left\{\frac{\sup_{t\in\mathbb{R}} e^{\sqrt{2}B_\alpha(t)-|t|^\alpha}}{\int_{t\in\mathbb{R}} e^{\sqrt{2}B_\alpha(t)-|t|^\alpha}\,dt}\right\},$$

with $\{B_\alpha(t), t \in \mathbb{R}\}$ a standard fractional Brownian motion (fBm) with Hurst index $\alpha/2$, i.e., B_α is a centered self-similar Gaussian process with Hurst index $\alpha/2$, stationary increments and $\mathbb{E}\{B_\alpha^2(t)\} = |t|^\alpha, t \in \mathbb{R}$.
If X is a centered non-stationary Gaussian process with continuous sample paths and variance function $\sigma^2(t)$ that has a unique point of maximum in $[0, T]$, say, 0 with $\sigma(0) = 1$, then in view of [4], we have

$$\mathbb{P}(M(T) > u) \sim K_{\alpha,\beta}u^{\max(0,2/\alpha-2/\beta)}\mathbb{P}(X(0) > u), \tag{3}$$

provided that for $\alpha \in (0, 2], a > 0$, the correlation function $r(s, t)$ satisfies

$$1 - r(s, t) = a|t - s|^\alpha + \mathrm{o}(|t - s|^\alpha) \quad \text{as } s, t \to 0,$$

and for b, β positive

$$1 - \sigma(t) = b|t|^\beta + \mathrm{o}(|t|^\beta) \quad \text{as } t \to 0, \tag{4}$$

assuming additionally that X satisfies a global Hölder condition. Here $K_{\alpha,\beta} = 2\mathcal{H}_\alpha a^{1/\alpha}\Gamma(1/\beta + 1)b^{-1/\beta}$ with $\Gamma(\cdot)$ the Euler Gamma function, for $\alpha < \beta$; $K_{\alpha,\beta} = 1$ for $\alpha > \beta$ and $K_{\alpha,\alpha}$ equals the Piterbarg constant

$$\mathcal{H}_\alpha^{b/a} = \lim_{T\to\infty} \mathbb{E}\left\{\sup_{t\in[0,T]} e^{\sqrt{2}B_\alpha(t)-(1+b/a)t^\alpha}\right\} \in (0, \infty).$$

Date: August 22, 2017.