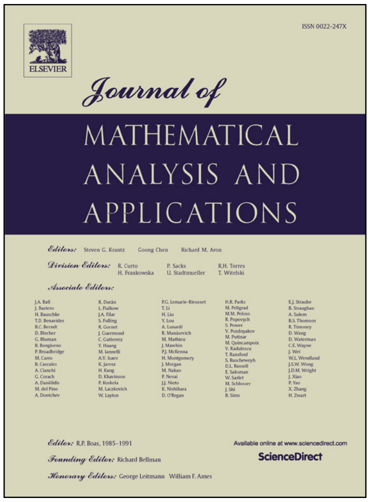

Travelling wave solutions for a nonlocal dispersal HIV infection dynamical model

Wei Wang, Wanbiao Ma

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Travelling wave solutions for a nonlocal dispersal HIV infection dynamical model ☆

Wei Wang, Wanbiao Ma*

Department of Applied Mathematics, School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China

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Abstract

This paper is devoted to developing a nonlocal dispersal HIV infection dynamical model. The existence of travelling wave solutions is investigated by employing Schauder's fixed point theorem. That is, we study the existence of travelling wave solutions for $R_0 > 1$ and each wave speed $c > c^*$. In addition, the boundary asymptotic behavior of travelling wave solutions at $+\infty$ is obtained by constructing suitable Lyapunov functions and employing Lebegue dominated convergence theorem. By employing a limiting argument, we investigate the existence of travelling wave solutions for $R_0 > 1$ and $c = c^*$. The main difficulties are that the semiflow generated by the model does not have the order-preserving property and the solutions lack of regularity.

Keywords: HIV, Nonlocal dispersal, Travelling wave solutions, Lyapunov functions, Schauder's fixed point theorem.

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1. Introduction and model derivation

In this paper, we consider the existence of travelling wave solutions of the following nonlocal dispersal HIV infection dynamical model with Beddington-DeAngelis functional response

$$\begin{cases} \dfrac{\partial U(x,t)}{\partial t} = D[(J*U)(x,t) - U(x,t)] + \zeta - dU(x,t) - \dfrac{\beta U(x,t)\omega(x,t)}{1 + aU(x,t) + b\omega(x,t)}, \\ \dfrac{\partial V(x,t)}{\partial t} = D[(J*V)(x,t) - V(x,t)] + \dfrac{\beta U(x,t)\omega(x,t)}{1 + aU(x,t) + b\omega(x,t)} - a_1 V(x,t), \\ \dfrac{\partial \omega(x,t)}{\partial t} = D[(J*\omega)(x,t) - \omega(x,t)] + kV(x,t) - \mu\omega(x,t), \end{cases} \tag{1.1}$$

where $U(x,t)$, $V(x,t)$ and $\omega(x,t)$ denote the densities of target cells, infected cells and virus at location x and time t, respective. ζ is the target cells production rate. Target cells become infected cells at rate $\beta U(x,t)\omega(x,t)/(1 + aU(x,t) + b\omega(x,t))$, where a determines how fast the infection rate approaches its saturation value, and b is a measure of virus interference during infection [1]. $kV(x,t)$ is the production of free viruses. d, a_1 and μ are the death rates of target cells, infected cells and free viruses, respectively. In addition,

$$J*U(x,t) = \int_R J(x-y)U(y,t)dy$$

is the rate at which the target cells are arriving at position x from all other places, and

$$-U(x,t) = -\int_R J(x-y)U(x,t)dy$$

is the rate at which they are leaving location x to travel to all other sites, where $J(x-y)$ is thought of as the probability distribution of jumping from location y to location x. Hence, $J*U(x,t) - U(x,t)$ describes that the

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*Corresponding author

Email addresses: `wei___wang@163.com` (Wei Wang), `wanbiao_ma@ustb.edu.cn` (Wanbiao Ma)