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The topological structure of complete noncompact submanifolds in spheres

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ABSTRACT

In this paper, we first show that δ-super stable complete noncompact minimal submanifolds in S^{m+n} or R^{m+n} with $\delta > (\frac{m-1}{m})^2$ admit no nontrivial L^2 harmonic 1-forms and have only one nonparabolic end, which generalizes Cao–Shen–Zhu's result in [2] on stable minimal hypersurface in R^{m+1} and Lin's result in [13] on $\frac{m-1}{m}$-super stable minimal submanifolds in R^{m+n}. Second, we prove that the dimension of the space of L^2 harmonic p-forms on M^m is zero or finite and there is only one nonparabolic end or finitely many nonparabolic ends of M under the assumptions on the Schrödinger operators involving the squared norm of the traceless second fundamental form.

1. Introduction

The geometric structure and topological properties of submanifolds in various ambient space have been studied extensively during past few years. In [2], Cao, Shen and Zhu showed that a complete connected stable minimal hypersurface in Euclidean space must have exactly one end. Its strategy was to utilize a result of Schoen–Yau asserting that a complete stable minimal hypersurface in Euclidean space can not admit a non-constant harmonic function with finite integral [16]. Later Ni [15] proved that if n-dimensional complete minimal submanifold M in Euclidean space has sufficient small total scalar curvature (i.e. $\int_M |A|^n < C_1$), then M has only one end. In [17], Seo improved the upper bound C_1. Due to this connection with harmonic functions, this allows one to estimate the number of ends of the above submanifold by estimating the dimension of the space of bounded harmonic function with finite Dirichlet integral (cf. [11]). In [5], Fu and Xu proved that a complete submanifold M^m with finite total curvature and some conditions on mean curvature in an $(n + p)$-dimensional simply connected space form $M^{m+p}(c)$ must have finitely many ends. In [4], M.P. Cavalcante, H. Mirandola, F. Vitório proved that a complete submanifold M^m with finite total curvature and some conditions on the first eigenvalue of the Laplace–Beltrami operator of M in an Hadamard manifold must have finitely many ends. In [13], H.L. Lin proved a vanishing and finiteness theorem for L^2

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harmonic forms under the assumptions on Schrödinger operators involving the squared norm of the traceless second fundamental form. In [14], H.Z. Lin obtained some vanishing theorems for L^2 forms on hypersurfaces in sphere. In [18,19], P. Zhu and S.W. Fang obtained some vanishing and finiteness theorems for L^2 harmonic 1-forms on submanifolds in spheres.

In general, one is interested in understanding relations between the topology and geometry of a Riemannian manifold M and the spaces of harmonic forms. When M is compact, by Hodge theory, the space of harmonic p-forms on M is isomorphic to its p-th de Rham cohomology group. When M is noncompact, the Hodge theory does not work anymore, and it is natural to consider L^2 forms, as it is showed that L^2 Hodge theory remains valid in noncompact manifolds as classical Hodge theory works well in the compact case. For more results concerning L^2 harmonic forms on complete noncompact manifolds, one can consult a very nice survey of Carron [3].

In this paper, we investigate the relations between the index of Schrödinger operators of submanifolds M^m in S^{m+n} or R^{m+n} and the space of $L^{2\beta}$ harmonic p-forms on them and the number of their ends. We assume that M^m is a complete noncompact manifold and define the space of the $L^{2\beta}$ harmonic p-forms on M by

$$H^p(L^{2\beta}(M)) = \{\omega | \int_M |\omega|^{2\beta} < \infty, \triangle\omega = 0\}.$$

We obtain the following results.

Theorem 1.1 *(cf. Theorem 3.1). Let M^m $(m \geq 3)$ be an m-dimensional complete noncompact oriented manifold isometrically immersed in an $(m+n)$-dimensional sphere S^{m+n}. Assume that M^m is a minimal submanifold in S^{m+n} and $\lambda_1(\triangle + \delta|A|^2) \geq 0$ for $1 \leq p \leq m-1$. When $2 \leq p \leq m-2$, assume further that M^m has flat normal bundle. Then $H^p(L^{2\beta}(M)) = \{0\}$, where δ and β are constants satisfying the following inequalities:*

$$\delta > \frac{p(m-p)}{m(1+K_p)}$$

and

$$1 \leq \beta < \frac{m\delta}{p(m-p)} + \frac{m\delta}{p(m-p)}\sqrt{(\frac{p(m-p)}{m\delta} - 1)^2 + \frac{p(m-p)}{m\delta}(K_p + 1 - \frac{p(m-p)}{m\delta})}.$$

Remark 1.2. Let F be a function defined by $F(x) = \frac{1}{x} + \frac{1}{x}\sqrt{(x-1)^2 + x(K_p + 1 - x)}$, where $0 < x < K_p + 1$. When $1 \leq x < K_p + 1$, we have $F(x) > \frac{1}{x} + \frac{1}{x}\sqrt{(x-1)^2} = 1$. When $0 < x < 1$, we have $F(x) > \frac{1}{x} > 1$. So we have $F(x) > 1$ for any $x \in (0, K_p + 1)$. Now we take $x = \frac{p(m-p)}{m\delta}$, so we can obtain that $x \in (0, K_p + 1)$ and $F(\frac{p(m-p)}{m\delta}) > 1$.

Corollary 1.3. *Let M^m $(m \geq 3)$ be an m-dimensional complete noncompact minimal submanifold immersed in an $(m+n)$-dimensional sphere S^{m+n}. If M is δ-super-stable with $\delta > (\frac{m-1}{m})^2$, then $H^1(L^2(M)) = \{0\}$ and M has only one nonparabolic end.*

By using a similar method as in the proof of Theorem 1.1, we can obtain the following results.

Theorem 1.4. *Let M^m $(m \geq 3)$ be an m-dimensional complete noncompact minimal submanifold immersed in R^{m+n}. If M is δ-super-stable with $\delta > (\frac{m-1}{m})^2$, then $H^1(L^2(M)) = \{0\}$ and M has only one nonparabolic end.*

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