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Identification of the initial condition in backward problem with nonlinear diffusion and reaction

Vo Van Au [a,b], Nguyen Huy Tuan [c,*]

[a] Department of Mathematics and Computer Science, University of Natural Science, Vietnam National University at Ho Chi Minh City, Viet Nam
[b] Faculty of General Sciences, Can Tho University of Technology, Can Tho City, Viet Nam
[c] Faculty of Mathematics and Statistics, Ton Duc Thang University, Ho Chi Minh City, Viet Nam

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ABSTRACT

In this paper, we study the backward in time problem for nonlinear parabolic equations associated with nonlinear diffusion coefficient. The problem is ill-posed in the sense of Hadamard. Under an a priori assumption on the exact solution in Gevrey space, we propose a new regularization method for stabilising the ill-posed problem. Our results extend some earlier works on backward problems for nonlinear parabolic equations.

1. Introduction

Let T be a positive number and $\Omega \subset \mathbb{R}^n, n \geq 1$ be an open bounded domain with a smooth boundary Γ. Set $Q = \Omega \times (0,T), \Sigma = \Gamma \times (0,T)$; Σ is called the lateral boundary of the cylinder Q. In this paper, we consider the question of finding the function $u(x,t)$, $(x,t) \in \overline{\Omega} \times [0,T]$, satisfying the problem

$$\begin{cases} u_t - \nabla \cdot \Big(\mathcal{L}(x,t;u)\nabla u\Big) = \mathcal{F}(x,t;u), & \text{in } Q, \\ u = 0, & \text{on } \Sigma, \\ u(x,T) = \chi(x), & \text{in } \Omega, \end{cases} \tag{1.1}$$

where χ is given function (usually in $L_2(\Omega)$), the function $\mathcal{F}$ is the source term and $\mathcal{L}(x,t;u)$ is the positive diffusion coefficient (or a positive definite and symmetric matrix) which may depend on not only the independent variables (x,t) but also on the dependent variable u. In practice, we get the data χ at discrete nodes so as to contain measurement errors in the form of the noisy data $\chi_\delta \in L_2(\Omega)$ satisfying

* Corresponding author.
E-mail addresses: vvau@ctuet.edu.vn (V.V. Au), nguyenhuytuan@tdt.edu.vn (N.H. Tuan).

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$$\|\chi_\delta - \chi\| \le \delta, \tag{1.2}$$

where the constant $\delta > 0$ represents a bound on the measurement error. This problem is well known to be ill-posed and regularization methods for it are required. For $\mathcal{F} = 0, \mathcal{L} = 1$, the instability grows exponentially with T.

Many papers are devoted to special cases of problem (1.1). The case $u_t = \mathcal{L}u_{xx}$ with constant $\mathcal{L} > 0$ has been investigated in many papers, [3,4,6–9,11,12,14–17,19,20,22,21,27,28]. The case $u_t = \mathcal{L}(t)u_{xx} + \mathcal{F}(x,t)$ has been considered in [19,25]. Tuan [24] investigated a regularization method and computation for the nonlinear case $u_t = \mathcal{L}(t)u_{xx} + \mathcal{F}(x,t;u)$. Very recently, Xu et al. [10] applied the modified-quasi boundary value method to regularize problem (1.1) with time dependent variable coefficient $\mathcal{L}(x,t) = \mathcal{L}(t)$.

To the best of our knowledge, there are not any results devoted to backward parabolic equations in the case of the coefficient $\mathcal{L}(x,t;u)$. In the more simple case, such as $\mathcal{L}(x,t;u) = \mathcal{L}(x,t)$, the first paper devoted to the homogeneous problem of (1.1) (i.e. $\mathcal{F} = 0$) seems to be that of Krein [13] and then further investigation by Hao and Duc [8]. The paper [8] introduced the method of non-local boundary value problems (or called quasi-boundary value method) to give the following regularized problem

$$\begin{cases} v_t^\delta - \nabla \cdot \Big(\mathcal{L}(x,t)\nabla v^\delta\Big) = 0, & \text{in } Q, \\ v^\delta = 0, & \text{on } \Sigma, \\ v^\delta(x,T) + \delta v^\delta(x,0) = \chi_\delta(x), & \text{in } \Omega. \end{cases} \tag{1.3}$$

By using the log-convexity method, they gave convergence rates between the sought solution of homogeneous problem and the regularized solution v^δ of (1.3) with some a priori conditions for the sought solution u, i.e. $u(\cdot,0) \in L_2(\Omega)$.

As is known, if the coefficient $\mathcal{L}(x,t) = \mathcal{L}(t)$ or $\mathcal{L}(x,t) = \mathcal{L}(x)$ then problem (1.1) is equivalent to a nonlinear integral equation, [18,23,24]. However, for the general form of $\mathcal{L}(x,t;u)$, the problem (1.1) cannot be transformed into a nonlinear integral equation. Hence, some classical methods and previous techniques which used spectral methods are not applicable to approximating the problem (1.1). The nonlinear case is more difficult and a new method is required.

Our major object in this paper is to provide a new quasi-reversibility method for regularizing the nonlinear problem (1.1). We also estimate the error between the solution of (1.1) and that of (2.7) in the cases of the reaction term being a global or local Lipschitz function. The paper is organized as follows. In Section 2, a stability estimate is proved under an a priori condition on the exact solution and the globally Lipschitz source term. In Section 3, the analysis is extended to local Lipschitz source functions.

2. Backward problem with global Lipschitz reaction

When Ω is bounded, problem (1.1) can also be solved by a decomposition in a Hilbert basis of $L_2(\Omega)$. For this purpose, it is very convenient to choose a basis $\{\xi_i(x)\}_{i\in\mathbb{N}^*}$ of $L_2(\Omega)$ composed of eigenfunctions of $-\Delta$ (with zero Dirichlet condition), i.e.,

$$\begin{cases} -\Delta\xi_i(x) = \lambda_i\xi_i(x), & \text{in } \Omega, \\ \xi_i(x) = 0, & \text{on } \Gamma, \end{cases} \tag{2.4}$$

which admits a family of eigenvalues $0 < \lambda_1 \le \lambda_2 \le \lambda_3 \le \ldots \le \lambda_i \le \ldots$ and $\lambda_i \to \infty$ as $i \to \infty$ (see [5], p. 335).