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Optimal design of fractured media with prescribed macroscopic strain

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ABSTRACT

In this work we consider an optimal design problem for two-component fractured media for which a macroscopic strain is prescribed. Within the framework of structured deformations, we derive an integral representation for the relaxed energy functional. We start from an energy functional accounting for bulk and surface contributions coming from both constituents of the material; the relaxed energy densities, obtained via a blow-up method, are determined by a delicate interplay between the optimization of sharp interfaces and the diffusion of microcracks. This model has the far-reaching perspective to incorporate elements of plasticity in optimal design of composite media.

1. Introduction

Starting with the pioneering papers by Kohn and Strang [19–21], much attention has been drawn to optimal design problems for mixtures of conductive materials. The variational formulation of these problems, particularly useful for finding configurations of minimal energy, entails some technical problems from the mathematical point of view, in particular the non-existence of solutions. In [3,18] this issue is addressed by introducing a perimeter penalization in the energy functional to be minimized, which has also the effect of discarding configurations where the two materials are finely mixed. For a related problem, leading to a similar energy functional in the context of brutal damage evolution, see [1] and [14].

In the spirit of [22,23] we want to study an optimal design problem which can incorporate elements of plasticity, in a way that it is suited to treat both composite materials (made of components with different

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mechanical properties) and polycrystals (where the same material develops different types of slips and separations at the microscopic level). In order to do so, we extend the framework introduced in [3], by considering a material with two components each of which undergoes an independent (first-order) *structured deformation*, according to the theory developed by Del Piero and Owen [12]. The generalization of our model to account for materials with more than two components, or to polycrystals, is straightforward.

Structured deformations set the basis to address a large variety of problems in continuum mechanics where geometrical changes can be associated with both classical and non-classical deformations for which an analysis at macroscopic and microscopic level is required. For instance, in a solid with a crystalline defective structure, opening of cracks at the macroscopic level may compete with slips and lattice distortions at the microscopic level preventing the use of classical theories, where deformations are assumed to be smooth. The objective of the theory of structured deformations is to generalize the theoretical apparatus of continuum mechanics as a starting point for a unified description of bodies with microstructure. It also turns out to be relevant to describe phenomena as plasticity, damage, creation of voids, mixing, and fracture in terms of the underlying microstructure (see [12]).

We discuss now in more detail the application to polycrystals, which consist of a large number of grains, each having a different crystallographic orientation, and where the intrinsic elastic and plastic response of each portion may vary from point to point. The anisotropic nature of crystal slip usually entails reorientation and subdivision phenomena during plastic straining of crystalline matter, even under homogeneous and gradient-free external loadings. This leads to spatial heterogeneity in terms of strain, stress, and crystal orientation. Beyond the aim of gaining fundamental insight into polycrystal plasticity, an improved understanding of grain-scale heterogeneity is important, and this is the main motivation for our work. As noted in [25], structural and functional devices are increasingly miniaturized. This involves size reduction down to the single crystal or crystal-cluster scale. In such parts, crystallinity becomes the dominant origin of desired or undesired anisotropy. In miniaturized devices plastic heterogeneity and strain localization can be sources of quality loss and failure. Thus, optimized design of small crystalline parts requires improved insight into crystal response and kinematics at the grain and subgrain scale under elastic, plastic, or thermal loadings. Moreover, the better understanding of the interaction between neighboring grains, namely the quantification of its elastoplastic interaction, is in itself relevant for the verification and improvement of existing polycrystals homogenization models. These models are often considered to capture the heterogeneities on material response for polycrystals, see, e.g., [24]. In this spirit, this work can also be viewed as a first step towards the derivation of a homogenization result for a polycrystalline material in the context of plasticity.

To minimize our functional from a variational point of view, we rely on the energetics for structured deformations first studied by Choksi and Fonseca [9], where the problem is set in the space of special functions of bounded variation. Given an open bounded subset $\Omega \subset \mathbb{R}^N$, a structured deformation (in the context of [9]) is a pair $(g, G) \in SBV(\Omega;\mathbb{R}^d)\times L^1(\Omega;\mathbb{R}^{d\times N})$, where g is the microscopic deformation and G is the macroscopic deformation gradient. The energy associated with a structured deformation is then defined as the most effective way to build up the deformation using sequences $u_n \in SBV(\Omega;\mathbb{R}^d)$ that approach (g, G) in the following sense: $u_n \to g$ in $L^1(\Omega;\mathbb{R}^d)$ and $\nabla u_n \rightharpoonup G$ in $L^p(\Omega;\mathbb{R}^{d\times N})$, for $p > 1$ a given summability exponent. The convergences above imply that the singular parts $D^s u_n$ converge, in the sense of distributions, to $Dg - G$. To have a better understanding of this phenomenon, consider the simpler case of a deformation $g \in W^{1,1}(\Omega;\mathbb{R}^d)$, that is, without macroscopic cracks. Then, $Du_n = \nabla u_n \mathcal{L}^N + D^s u_n$ with $D^s u_n$ absolutely continuous with respect to the Hausdorff measure $\mathcal{H}^{N-1}$ and supported in $S(u_n)$, the jump set of u_n. Since $Du_n \rightharpoonup \nabla g$ in the sense of distributions and $\nabla u_n \rightharpoonup G$, we conclude that $D^s u_n \rightharpoonup \nabla g - G$ in the sense of distributions.

This tells us that the difference between microscopic and macroscopic deformations is achieved through a limit of singular measures supported in sets $S(u_n)$ such that $\mathcal{H}^{N-1}(S(u_n)) \to +\infty$. The tensor $M := \nabla g - G$ is called the *disarrangements tensor* and embodies the fact that the difference between the microscopic and the macroscopic deformations in the bulk are achieved as a limit of singular measures.

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