

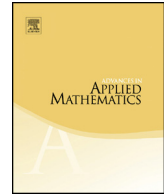


ELSEVIER

Contents lists available at ScienceDirect

Advances in Applied Mathematics

www.elsevier.com/locate/yaama



Profinite automata



Eric Rowland^{a,b,1}, Reem Yassawi^{c,d}

^a *University of Liege, Belgium*

^b *Hofstra University, Hempstead, NY, USA*

^c *Trent University, Peterborough, Canada*

^d *IRIF, CNRS UMR 8243, Université Paris-Diderot, France*

ARTICLE INFO

Article history:

Received 8 March 2015

Received in revised form 16

November 2016

Accepted 17 November 2016

Available online xxx

MSC:

37B10

11B85

11A07

05A15

ABSTRACT

Many sequences of p -adic integers project modulo p^α to p -automatic sequences for every $\alpha \geq 0$. Examples include algebraic sequences of integers, which satisfy this property for every prime p , and some cocycle sequences, which we show satisfy this property for a fixed p . For such a sequence, we construct a profinite automaton that projects modulo p^α to the automaton generating the projected sequence. In general, the profinite automaton has infinitely many states. Additionally, we consider the closure of the orbit, under the shift map, of the p -adic integer sequence, defining a shift dynamical system. We describe how this shift is a letter-to-letter coding of a shift generated by a constant-length substitution defined on an uncountable alphabet, and we establish some dynamical properties of these shifts.

© 2016 Elsevier Inc. All rights reserved.

E-mail address: eric.rowland@hofstra.edu (E. Rowland).

¹ Supported by a Marie Curie Actions COFUND fellowship.

1. Introduction

A *substitution* (or *morphism*) on an alphabet \mathcal{A} is a map $\theta : \mathcal{A} \rightarrow \mathcal{A}^*$, extended to $\mathcal{A}^{\mathbb{N}}$ by concatenation. A substitution is *length- k* (or *k -uniform*) if, for each $a \in \mathcal{A}$, the length of $\theta(a)$ is k . The extensive literature on substitutions has traditionally focused on the case where \mathcal{A} is finite. Some exceptions include recent work, for example in [12] and [14]. Substitutions on a countably infinite alphabet have been used to describe lexicographically least sequences on \mathbb{N} avoiding certain patterns [13,19], and they have been used in the combinatorics literature to enumerate permutations avoiding patterns [25].

In this article we present a new construction for constant-length substitutions on an uncountable alphabet. Our motivation comes from the following classical results. Let $(a(n))_{n \geq 0}$ be an *automatic sequence* (see Definition 2.2). Cobham's theorem (Theorem 2.6) characterizes an automatic sequence as the coding, under a letter-to-letter map, of a fixed point of a constant-length substitution. Christol's theorem (Theorem 2.8) characterizes p -automatic sequences for prime p ; they are precisely the sequences whose generating function is algebraic over a finite field of characteristic p . The following can be viewed as a generalization of one direction of Christol's characterization.

Theorem 1.1 ([6, Theorem 32], [10, Theorem 3.1]). *Let $(a(n))_{n \geq 0}$ be a sequence of p -adic integers such that $\sum_{n \geq 0} a(n)x^n$ is algebraic over $\mathbb{Z}_p(x)$, and let $\alpha \geq 0$. Then $(a(n) \bmod p^\alpha)_{n \geq 0}$ is p -automatic.*

Thus certain p -adic integer sequences (and, in particular, integer sequences) have the property that they become p -automatic when reduced modulo p^α , for every $\alpha \geq 0$. More generally, the diagonal of a multivariate rational power series is p -automatic when reduced modulo p^α , and one can explicitly compute an automaton for $(a(n) \bmod p^\alpha)_{n \geq 0}$ for all but finitely many primes p [20, Theorem 2.1].

Fix a prime p , and let $(a(n))_{n \geq 0}$ be a sequence such that $(a(n) \bmod p^\alpha)_{n \geq 0}$ is p -automatic for every $\alpha \geq 0$. For each α , there is a finite automaton generating $(a(n) \bmod p^\alpha)_{n \geq 0}$. In Lemma 3.1 we show that these automata can be chosen in a compatible way; namely, their inverse limit exists. In this way we obtain a *profinite automaton* (Definition 3.3) generating the sequence $(a(n))_{n \geq 0}$.

We can obtain other inverse limit objects from a p -adic integer sequence in a similar way. In particular, Cobham's theorem guarantees a length- p substitution θ_α such that $(a(n) \bmod p^\alpha)_{n \geq 0}$ is a coding of a fixed point of θ_α . Each substitution θ_α is a substitution on a finite alphabet, but their inverse limit is a profinite substitution on an alphabet that is, in general, uncountable (Theorem 4.2). This alphabet has a natural coding to the set \mathbb{Z}_p of p -adic integers, and the sequence $(a(n))_{n \geq 0}$ is the coding of a fixed point of the profinite substitution.

With this profinite substitution, we obtain a shift (Theorem 4.1), as in the classical finite-alphabet case. This shift is the closure of the orbit, under the shift map, of a fixed point (or coding of a fixed point) of the profinite substitution. One feature of profinite

Download English Version:

<https://daneshyari.com/en/article/5775435>

Download Persian Version:

<https://daneshyari.com/article/5775435>

[Daneshyari.com](https://daneshyari.com)