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# Synchronization of nonlinear complex dynamical systems via delayed impulsive distributed control

Huilan Yang[a], Xin Wang[b,*], Shouming Zhong[a], Lan Shu[a]

[a] School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China
[b] School of Information and Software Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, PR China

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ABSTRACT

This paper investigates the exponential synchronization problem of nonlinear complex dynamical systems via delayed impulsive distributed control. Different from the existing results on the synchronization of complex dynamical systems, impulsive input delays are considered in our model. Combined with the time-varying Lyapunov functional and mathematical induction approaches, criteria on system synchronization are established, which sufficiently utilize the information of both the state variables of themselves and their neighbors. Moreover, it is shown that the frequency of impulsive occurrence and impulsive input delays can heavily affect the synchronization performance. Finally, two numerical simulations are given to illustrate the effectiveness of the derived theoretical results.



## 1. Introduction

During the past decades, complex dynamical networks (CDNs) have been widely used to model and study many real-world physical and social systems, from the internet to world wide web, from communication networks to social organizations, from food webs to ecological communications and so forth [1–3]. In particular, as a kind of typical collective behaviors exhibited in CDNs, synchronization has received considerable attention in many practical fields including information science, parallel image processing, secure communication, and neural networks [4–10].

Much effort has been made to develop effective control strategies to regulate the network into a synchronous motion, such as continuous feedback control, adaptive control, impulsive control [11] and sampled-data control [12]. Among them, impulsive control is regarded as an effective control method which allows systems to possess discontinuous inputs. As a result, this scheme provides a promising prospect for solving issues in which systems cannot endure continuous control inputs, or in some practical applications it is not possible to support continuous control inputs. Consequently, in this regard, impulsive control method may be the better choice and has been widely used to synchronize CDNs and chaotic systems. Recently, many researchers have investigated the synchronization problem of CDNs with impulsive control [13–15]. For example, in [14], Yang, et al., studied the synchronization problem of complex networks with nonidentical nodes via hybrid adaptive and impulsive controls. In [15], the synchronization issue of nonlinear dynamical networks by pinning impulsive strategy was analyzed.

* Corresponding author.
*E-mail address:* xinwang201314@126.com (X. Wang).

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Moreover, when we model the real-world CDNs, time delays are necessary to be taken into account due to the finite speed of switching and transmitting signals, which may result in oscillatory behavior or network instability. Hence the related results on the synchronization issue of delayed complex dynamical networks (DCDNs) have been extensively reported [16–22]. In general, there are two types of time delays in dynamical networks. One is internal delay occurring inside the dynamical node. The other is coupling delay caused by the exchange of information between dynamical nodes. In this paper, the problem of synchronization for complex networks with both the internal delay and coupling delay will be considered simultaneously. On the other hand, delayed impulses, it is regarded as a better way to model many practical problems [23–26]. For example, in communication security systems [27,28], due to the limit speed of signal sampling, a type of delays called sampling delays which depend on the historical states at the sampling points will occur in impulsive transients. Another example, in population dynamics such as fishing industry [29], effective impulsive control such as harvesting and releasing can keep the balance of fishing, and the quantities of every impulsive harvesting or releasing are not only measured by the current numbers of fish but also depend on the numbers in recent history since the fact that the immature fish need some time to grow. Besides, in some other models arising from digital communication, neural networks and ecological models [30–36], delayed impulses have more potential applications.

However, it is worth noting that all the results mentioned above on the synchronization of complex networks with impulsive control are more suitable to the CDNs with weak coupling. As was shown in [37], this is because the node on the impulsive controller can not exchange the state information with their neighboring nodes in these decentralized impulsive control schemes. Thus, when the inner coupling is not weak, a distributed control approach may be needed. For example, in [38], a distributed impulsive control scheme was proposed to achieve synchronization for a class of DCDNs. Since this idea uses the information from a wider subset of the network nodes, it may assure synchronization of the DCDNs with stronger coupling than decentralized impulsive control. Recently, some research efforts have been devoted to the distributed impulsive control and several kinds of novel distributed control methods have been constructed in [39–41]. For instance, in [40], the synchronization issue of unknown nonlinear networked systems via distributed adaptive control was addressed. In [41], the leader-follower synchronization problem was considered for heterogeneous dynamical networks via distributed impulsive control. However, they did not take into account the impulsive input delays. In addition, how to design the impulse sequences was also not mentioned. Thus, an interesting question is whether we can take into consideration of impulse delays in the distributed impulsive controller to achieve synchronization of CDNs?

Motivated by the above discussions, this paper studies the problem of exponential synchronization for CNDs under the delay-dependent impulsive distributed control. The main contributions of this paper are listed as follows: (1), this is the first time that a delayed distributed impulsive controller is proposed to achieve the network synchronization. (2), by employing the time-varying Lyapunov functional and mathematical induction approaches, sufficient criteria on network synchronization are established, which sufficiently utilizes the information of both the state variables of themselves and their neighbors. (3), the derived criteria reveal that the frequency of impulsive occurrence and impulsive input delays can heavily affect the synchronization performance. Finally, two numerical simulations are given to illustrate the effectiveness of the derived theoretical results.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes $n$-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $\mathbb{N}=\{0,1,2,\ldots\}$. For a real symmetric matrices $P$, the notation $P>0(P<0)$ means that matrix $P$ is positive(negative) definite. $I$ and $I_n$ denote an identity matrix with appropriate dimension and the $n\times n$ real identity matrix, respectively. For $x\in\mathbb{R}^n$, $x^T$ denotes its transpose. The vector norm is defined as $\|x\|=\sqrt{x^Tx}$. For matrix $A\in\mathbb{R}^{n\times n}$, $\|A\|=\sqrt{\lambda_{max}(A^TA)}$, where $\lambda_{max}(\cdot)$ represents the largest eigenvalue. $PC([-\bar{\tau},0];\mathbb{R}^n)$ denotes the family of piecewise continuous functions from $[-\bar{\tau},0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|_{\bar{\tau}}=sup_{-\bar{\tau}\le s\le 0}|\phi(s)|$. $\mathcal{N}=\{1,2,\ldots,N\}$.

## 2. Problem statement and preliminaries

In this paper, we consider a complex network consisting of $N$ linearly delayed coupled identical nodes, which is described by

$$\dot{x}_i(t)=Ax_i(t)+f(t,x_i(t),x_i(t-\tau(t)))+c\sum_{j=1}^{N}g_{ij}\Gamma x_j(t-\tau(t))+u_i(t),\quad i\in\mathcal{N}. \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbb{R}^n$ is the state variable of the $i$th node, $u_i(t)$ is the control input. $f(t,x_i(t),x_i(t-\tau(t)))=(f_1(t,x_i(t),x_i(t-\tau(t))),f_2(t,x_i(t),x_i(t-\tau(t))),\ldots,f_n(t,x_i(t),x_i(t-\tau(t))))^T\in\mathbb{R}^n$ is a continuous function. $c$ is the coupling strengthen, $\Gamma\in\mathbb{R}^{n\times n}$ is an inner-coupling matrix. $\tau(t)$ is the time-varying coupling delay satisfying $0\le\tau(t)\le\tau$ for positive scalar $\tau$. $G=(g_{ij})_{N\times N}$ represents the outer-coupling configuration of the network. In which $g_{ij}$ is defined as follows: if there is a link from node $j$ to node $i(i\neq j)$, then $g_{ij}>0$; otherwise, $g_{ij}=0$. At the same time, the diagonal elements of $G$ is defined as $g_{ii}=-\sum_{j=1,j\neq i}^{N}g_{ij}$. Let $\mathcal{C}=\{(i,j):g_{ij}>0,i,j\in\mathcal{N}\}$ and $\mathcal{N}_i=\{j\in\mathcal{N}:(i,j)\in\mathcal{C}\}$.

In the following, some basic definition, lemmas and assumptions will be given.

**Definition 1.** The complex dynamical network (1) is said to achieve globally exponential synchronization, if there exist constants $M>0$ and $\varepsilon>0$ such that for any initial conditions the following inequality

$$\|x_i(t)-x_j(t)\|\le Me^{-\varepsilon(t-t_0)}$$