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Random neighborhood graphs as models of fracture networks on rocks: Structural and dynamical analysis

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ABSTRACT

We propose a new model to account for the main structural characteristics of rock fracture networks (RFNs). The model is based on a generalization of the random neighborhood graphs to consider fractures embedded into rectangular spaces. We study a series of 29 real-world RFNs and find the best fit with the random rectangular neighborhood graphs (RRNGs) proposed here. We show that this model captures most of the structural characteristics of the RFNs and allows a distinction between small and more spherical rocks and large and more elongated ones. We use a diffusion equation on the graphs in order to model diffusive processes taking place through the channels of the RFNs. We find a small set of structural parameters that highly correlates with the average diffusion time in the RFNs. We found analytically some bounds for the diameter and the algebraic connectivity of these graphs that allow to bound the diffusion time in these networks. We also show that the RRNGs can be used as a suitable model to replace the RFNs in the study of diffusion-like processes. Indeed, the diffusion time in RFNs can be predicted by using structural and dynamical parameters of the RRNGs. Finally, we also explore some potential extensions of our model to include variable fracture apertures, the possibility of long-range hops of the diffusive particles as a way to account for heterogeneities in the medium and possible superdiffusive processes, and the extension of the model to 3-dimensional space.

1. Introduction

It could be argued that in any system transporting mass and energy there should be an underlying network responsible for conducting the materials through space. The flow of fluids of petrochemical interest obeys this general rule, where the role of the transporting network is played by the system of rock fractures. The study of rock fracture systems have a long tradition in hydrocarbon geology and hydrogeology due to the role that these fractures play on the evaluation of potential oil reservoirs (Adler and Thovert [1], Andresen et al. [4], Cacas et al. [9], Hitchmough et al. [28], Huseby et al. [29], Valentini et al. [50]). The analysis of rock fracture networks (RFNs) plays a fundamental role in determining the nature and disposition of heterogeneities appearing in petroliferous formations to determine the capability for the transport of fluid through them (Bogatkov and Babadagli [7], Han et al. [24], Hansford and Fisher [25], Jang et al. [31], Wilson et al. [53]). In many of the analyses described in the literature the use of synthetic fracture networks facilitates the analysis due to the sometimes scarce availability of real-world data (Barton [5], Damjanac and Cundall [11], Garipov et al. [23], Neuman [38], Nolte et al. [40], Seifollahi et al. [46], Xu and Dowd [54]). In contrast, Santiago et al. have published a series of papers (Santiago et al.

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[42–44]) in which they used real fracture networks derived from original hand-sampled images of rocks extracted from a Gulf of Mexico oil reservoir. These works have used a graph-theoretic analysis of these real-world networks in order to extract information about the topological (static) characteristics of this group of rocks. Rock fractures have also been studied in a more general sense for their applications to both oil reservoirs and other fluid flows within rocks such as groundwater, examining properties such as fractal scaling and anomalous diffusion (Berkowitz [6], Bonnet et al. [8], Edery et al. [12]).

This work is a step forward in the analysis of real rock fracture networks. First, a synthetic model reproducing the topology of real RFNs is proposed. This model, which is based on a generalization of the random neighborhood graphs (Jaromczyk and Toussaint [32], Toussaint [49]), is compared statistically with the real RFNs using a thorough topological characterization of the structure of these networks. The random neighborhood graphs represent a family of simple graphs in which two vertices are connected by an edge if and only if the vertices satisfy particular geometric requirements, and they involve a spatial distance. The Euclidean distance is most common choice and will be used here. They have found multiple applications in cases where spatial properties of graphs are required.

A discrete version of the diffusion equation is used to study the diffusion of a fluid through the channels of the real RFNs. The diffusion through the real RFNs is compared to diffusion on the synthetic model, showing that this random model reproduces not only the most important structural properties of the real networks but also their diffusive properties. As in the series of papers by Santiago et al. [42–44], two-dimensional cuts of rocks that show a fracture network embedded into the rock sides are considered. Then, a potential criticism to these works is the fact that rock networks are three-dimensional (Andersson and Dverstorp [3], Koike et al. [34], Long and Billaux [36]) and that inferring these 3-dimensional networks from 2-dimensional information is hard. However, as has been previously documented, the analysis of 2-dimensional rock fractures identifies important parameters that allow the characterization of real rock samples (Jafari and Babadagli [30], Santiago et al. [42–44], Sarkar et al. [45]). In addition, note that the generalized proximity graphs that are introduced in this work can be easily extended to the 3-dimensional case. Thus, 3-dimensional rock fracture networks extrapolating the topological information that is obtained here from the analysis of 2-dimensional samples can be easily generated.

2. Rectangular β-skeleton graphs and relative neighborhood graphs

This section describes a generalization of the so-called β-skeleton graphs by considering not only points randomly distributed in a unit square but also in unit rectangles. The 'classical' β-skeleton graphs are described by Jaromczyk and Toussaint [32], Kowaluk [35], Toussaint [49]. The model can be briefly described as follows. Consider n points p_i $(i = 1, 2, \ldots, n)$ randomly and independently distributed in a unit square, and a value $\beta \geq 1$. Let p_i and p_j be two arbitrary points which are separated by a Euclidean distance L, and let $B(x, r)$ denote the circle located at x with radius r. For $\beta \geq 1$ the lune-based definition of the β-skeleton model is used and described here as it is more suitable for this work, an alternative is the circle-based definition which is not examined here. Two circles, $B((1-\frac{\beta}{2})p_i + \frac{\beta}{2}p_j, \frac{\beta}{2}L)$ and $B((1-\frac{\beta}{2})p_j + \frac{\beta}{2}p_i, \frac{\beta}{2}L)$ are constructed, and let R be intersection of the circles. It is obvious that the area of R increases as β is increased. Although it was previously stated that $\beta \geq 1$ for the sake of the current paper, the case of $0 < \beta < 1$ is also defined. In this case two circles of radius $L/(2\beta)$ which pass through both points p_i and p_j are instead constructed, and again the intersection is denoted by R, and note that the construction of the circles differs from the case of $\beta \geq 1$. Then, if there is no other point p_k included in the region R, the points p_i and p_j are connected by a segment of line, otherwise the points are not connected. By considering this process for all pairs of points, a graph $G = (V, E)$ is constructed in which the set of vertices V is formed by the points p_i and the set of edges E is formed by the segments of lines connecting pairs of vertices. Obviously, for small values of β, for example $0 < \beta < 1$, the chances that there is a point in the region R associated with p_i and p_j is very small, and there is a high probability that these two points are connected (see Fig. 1). As a consequence of this, the resulting graphs are very dense, containing a large number of triangles. It can be seen that if $\beta = 0$, the resulting graph is just the complete graph. Another particular case which is commonly considered in the computational geometry literature is when $\beta = 2$, which corresponds to the so-called relative neighborhood graph (RNG). Matlab code for creating such proximity graphs is provided in the Supplementary Information accompanying this paper.

2.1. Generalization of β-skeleton graphs

The generalization of the β-skeleton graphs is constructed by considering a rectangle $[0, a] \times [0, b]$ where $a, b \in \mathbb{R}$, $a \geq b$. Only unit rectangles of the form $[0, a] \times [0, a^{-1}]$ will be considered here. The rest of the construction of a rectangular β-skeleton graph is similar to that of a β-skeleton graph. That is, n points are distributed uniformly and independently in the unit rectangle $[0, a] \times [0, a^{-1}]$. Obviously, when $a = 1$ the rectangle $[0, a] \times [0, a^{-1}]$ is simply the unit square $[0, 1]^2$, which means that the rectangular β-skeleton graph becomes the classical β-skeleton one. Fig. 2 illustrates two rectangular β-skeleton graphs with $\beta = 2$ and different values of the rectangle side length a and the same number of nodes. In the first case when $a = 1$ the graph corresponds to the classical β-skeleton graph in which the nodes are embedded into a unit square. The second case corresponds to $a = 2$ which is a slightly elongated rectangle.