Applied Mathematics and Computation 316 (2018) 438–459

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Co-dynamics of Pneumonia and Typhoid fever diseases with cost effective optimal control analysis

Getachew Teshome Tilahun[a,*], Oluwole Daniel Makinde[b], David Malonza[c]

[a] Pan African University Institute of Basic Sciences Technology and Innovation, Nairobi, Kenya
[b] Faculty of Military Science, Stellenbosch University, South Africa
[c] Department of Mathematics, Kenyatta University, Kenya

ARTICLE INFO

Keywords:
Co-infection
Pneumonia
Typhoid fever
Optimal control
Cost effectiveness
Numerical simulation

ABSTRACT

In this paper, we propose and analyze a mathematical model for Pneumonia–Typhoid co-infection to examine their characteristic relationship due to preventive and treatments strategies. Firstly, we qualitatively analyzed the model, and the basic reproduction number is determined with respect to the existence and stability of equilibria. The possibility of bifurcation is studied and also we investigated the sensitivity index of co-infection model reproduction number together with the local and global stability of the equilibra. Secondly, we extend the co-infection model by incorporating time dependent controls as intervention and we used Pontryagin's maximum principle for derivation of necessary conditions for the optimal control and optimality system. Finally, the optimality system is numerically simulated by considering four strategies and also their cost effectiveness is analysed. We found that Pneumonia treatment with Typhoid fever prevention costs least. Therefore, for optimal cost effective control of both diseases, the policy makers must focus more on prevention strategy while treating the infected individual is not neglected.

1. Introduction

Pneumonia is classified as an airborne disease and most of the time it is acquired via inhalation or aspiration of pulmonary pathogenic organisms into a lung segments [1]. The disease can be characterized by an inflammatory condition of one or both of lungs. It can occur in all stages of human being but it is the most dangerous disease in older adults, babies, and people with other diseases or impaired immune systems [2]. Mainly Pneumonia is caused by bacteria and viruses. Fungi and parasites are also responsible for the causes of this disease. One of the most responsible cause of Pneumonia is called*Streptococcus Pneumoniae*, it is bacterial pneumonia.

Typhoid fever is a bacterial infection that can spread throughout the body affecting many organs. Without prompt treatment it can cause serious complications and can be fatal [3]. Typhoid fever is caused by virulent bacteria called *Salmonella typhi* (*S. typhi*).

* Corresponding author.
E-mail address: gmgech@gmail.com (G.T. Tilahun).

http://dx.doi.org/10.1016/j.amc.2017.07.063
0096-3003/

Mathematical modelling plays an important role in increasing our understanding of the dynamics of infectious diseases and also to investigate the optimal use of intervention strategies to control the spread of infectious diseases. A lot of scholars developed a mathematical model to describe the dynamics of the disease that helped them to propose disease control mechanism and also described the dynamics of the co-infection with other infectious diseases. Some of them are [4–7] proposed mathematical models that played significant roles in predicting suitable control strategies and analysing and ranking their cost-effectiveness strategies in mitigating the disease. Also, Akinyi et al. [8] developed a mathematical model for the impact of misdiagnosis and treatment of pneumonia as malaria. The result of the study showed that accurate diagnosis of both pneumonia and malaria can be reduced if the basic reproduction number is reduced below unity. David Onyinge and Omolo Ongati [6] proposed a mathematical model for the co-infection of Pneumonia–HIV/AIDS. Other studies [9], the authors developed a deterministic mathematical model for assessing the transmission dynamics of typhoid in malaria endemic settings. The result of the study suggests that a Typhoid fever outbreak in malaria endemic settings may lead to higher population of dually infected individuals displaying clinical symptoms of both infections than the singly-infected population displaying clinical symptoms of the diseases. Adeboye and Haruna [10] developed *SIRS* mathematical model that addressed the control of the transmission of Typhoid fever and Malaria simultaneously and also Akinyi et al. [11] propose a mathematical model for Malaria–Typhoid co-infection.

However, all of them did not consider optimal control strategies in their studies, and also there are few mathematical studies have been undertaken with co-infection of pneumonia with Malaria and also co-infection of Typhoid fever with Malaria. But still no work has been undertaken to describe co-dynamics of Pneumonia and Typhoid fever diseases with cost effective optimal control analysis to the best of our knowledge .

In this paper, an SIR model is formulated for Pneumonia–Typhoid fever co-infection with optimal problem and cost-effectiveness analysis of the optimal intervention strategies are presented. This paper is arranged as follows: in Section 2, we derive a model consisting of ordinary differential equations that describes the interactions and the dynamics of the diseases with the underlying assumptions. Section 3 is devoted to qualitative analysis of the model and sensitivity analysis of parameters. In Section 4, we use Pontryagin's s Maximum Principle to investigate optimal strategies and to find the necessary conditions for the optimal control of the disease. In Section 5, we show the simulation results. In Section 6 cost effectiveness analysis is described. Our conclusions are discussed in Section 7.

2. Model formulation

2.1. Description of the model

The model consider the Typhoid fever causing bacteria (salmonella) population (B) and the human population . The human population is divided in to seven classes, susceptible (S), Pneumonia infectious (I_p), Typhoid infectious (I_t), pneumonia and Typhoid co-infectious (I_{tp}), Pneumonia recovered (R_p), Typhoid recovered (R_t) and Pneumonia–Typhoid co-infectious recovered (R_{tp})is also considered. The recruitment rate susceptible individuals either by birth or immigration is π and the number of susceptible increases by those individuals that lost their temporary immunity from recovered sub classes of Pneumonia (R_p), Typhoid fever (R_t) and Pneumonia–Typhoid fever co-infected sub class (R_{tp}) with a rate of δ_1, δ_2 and δ_3 respectively. Any susceptible individuals either can get Pneumonia disease with force of infection $\lambda_p = \frac{\gamma(I_p+\Upsilon I_{tp})}{N}$ and join Pneumonia infectious sub-class (I_p) or Typhoid fever disease with force of infection $\lambda_t = \frac{\nu B}{k+B}$ and join Typhoid fever infectious sub-class(I_t), where $\Upsilon > 1$ is a modification parameter for the dually infected due to increased chance to infect susceptible than Pneumonia only infected. And also γ is infectious rate of Pneumonia, ν is the rate of ingestion of Typhoid causing bacteria, k is concentration of bacteria in foods and water. The number of co-infection sub- class is increased from Pneumonia infected group by getting Typhoid fever disease with λ_t force of infection and also from Typhoid fever infected sub-class by getting Pneumonia disease with λ_p force of infection. The infectious sub-class of Pneumonia also can get treatment with β_1 rate and move to Pneumonia recovered sub-class R_p or dies due to disease causing death rate of α_1. Similarly the infected sub-class of Typhoid fever also can get treatment with a rate of β_2 and join Typhoid recovered sub-class or dies from disease causing death with a rate of α_2. The Pneumonia- Typhoid co-infected sub class can get treatment with a rate of σ and get temporary immunity either from both disease or Pneumonia only or Typhoid only and join the co-infected recovered sub-class (R_{tp}) with probability of $(1-g)(1-e)$ or Pneumonia recovered sub-class (R_p) with probability of e or Typhoid fever recovered sub-class (R_t) with probability of $g(1-e)$, where $(1-g)(1-e)+g(1-e)+e=1$. Additionally, individuals in the co-infected sub class also dies either Pneumonia or Typhoid causing death with similar rate of Pneumonia or Typhoid only infected individuals. In all the seven human population sub-classes μ is natural causing death rate. The Typhoid fever causing bacteria (salmonella) population (B) grows in contaminated food or drinks with Q rate and also increase its number from the discharge of bacteria from Typhoid fever infected individuals and the co-infected individuals with a rate of σ_1 and σ_2 respectively and also it dies due to Natural/ drug induced death rate of μ_b.

The above description of the model is plotted in Fig. 1 and we generate the following model as seven system of differential equation.