Journal of Computational and Applied Mathematics 331 (2018) 11–22

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Linear estimation of physical parameters with subsampled and delayed data

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ARTICLE INFO

Article history:
Received 16 October 2016
Received in revised form 18 September 2017

MSC:
00-01
99-00

Keywords:
System identification
Parameter estimation
State-space models
Continuous-time models
Elasto-dynamics

ABSTRACT

An improved algorithm for the estimation of physical parameters with sub-sampled and delayed data is here presented. It shows a much better accuracy than the state-of-the-art when the sampling time of data acquisition T_s is much higher than the discretization step T_{sc} that should be used to get a highly accurate discrete model, i.e. $T_s \gg T_{sc}$, which is a common situation in multi-body and finite-element modelling applications. Moreover, the method proposed is capable of compensating delays between different acquisition channels. For the numerical experiments we focus on a mainstream class of models in applied mechanics, i.e. linear elasto-dynamics.

1. Introduction

There are, nowadays, a certain number of mechanical and mechatronic applications that require to estimate some physical parameter from the data collected in controlled experiments on a physical system. This is done traditionally off-line, with computer processing of these data. With current electronic devices these estimates may be conducted also in real-time on devices that act e.g. as embedded controllers of the system, and use these estimates for functional purposes; in this case these estimates are called indirect measurements, or *soft-sensors* and aim at the identification of a continuous-time model from experimental data [1,2].

The improved computer method here presented refers to the estimation through data collected in temporal transients; the data are collected in the time domain and also analysed in the same domain, using models that describe the temporal response dynamics of the system under test. In the literature there are also well-known techniques based on experiments with stationary signals, that involve vibration analysis (see e.g. [3] and contained citations), and estimate frequency responses of the system under test, through the analysis in the frequency domain of the experimental data.

First of all we will present a method that can estimate the physical parameters of a linear elasto-dynamic model using data sampled with a period T_s not too small and $T_s \gg T_{sc}$, where T_{sc} is the time-discretization step used to discretize the continuous model. This is a practical advantage and can be obtained simply by sampling with two different periods quite close each other, see Section 3; this is possible with the majority of the existing, even low end, electronic devices.

Moreover, we present a second improvement that allows to compensate unmodelled time-shifts between the collected signals. We will show that good results can be obtained by modifying the discretization algorithm from the best-practices used in the numerical simulation. For example, with the well-known θ-method [4], the choice of the parameter θ can be relevant and in realistic conditions (acquisition periods not very fast and I/O signals not perfectly synchronized) often it is

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https://doi.org/10.1016/j.cam.2017.09.036

better to choose its value different from the standard choices 0, 1 or 1/2 (Crank–Nicholson, also known as Tustin method or bilinear transform). In particular, this is the case when:

- The acquired signals are not perfectly synchronized, as is common in practice. In case of fast dynamics, even small time-shifts can alter significantly the estimated values. The method here proposed compensates for these shifts with a proper choice of θ, see Section 4.
- There is an unmodelled, small delay in the system response. For example, a transmission modelled as infinitely rigid but not so in practice, the delay introduced by some electronic device like DAC and ADC, omnipresent in acquisition systems and in actuators (mechanics is today more and more moved by electronic controllers and drivers), etc.

Note that the delay could be explicitly modelled in the system, at the price of a considerable extra-effort in the modelling phase and a complication in the numerical methods involved to discretize the continuous system. Such an approach [5] makes sense when the delay is reasonably big and it modifies substantially the response of a non-delayed system.

The paper is organized as follows. In Section 2 we describe the model problem and a common approach in parameter estimation. In Section 3 we present an improved linear estimation of physical parameters in case of $T_s \gg T_{sc}$ and in Section 4 an improved linear estimation of physical parameters in presence of delays. Section 5 contains some numerical experiments around the method proposed.

2. Problem statement and solution outline

Let us consider a generic linear elasto-dynamic model, represented by the following system of ordinary differential equations:

$$M\ddot{d}(t) + G\dot{d}(t) + Kd(t) = g(t), \tag{1}$$

where the matrices M, G, and K describe, respectively, the mass, damping and stiffness properties of the physical system, and $g(t)$ represents the forcing term (external forces). This model, although simple, is widely used in both continuum mechanics [6] and multibody modelling [7] and allows us to explain analytically the results obtained by the parameter estimation method here proposed.

Computer methods for this kind of model commonly are numerical integrations applied directly to model (1) or to its first-order-in-time reformulation. We choose this second form since, in this way, we translate the model (1) in the standard form used in system identification [1,8], that we will use for the estimation of the physical model parameters [3,9].

Therefore, let us reformulate the model (1) as a system of first-order, ordinary differential equations and add linear output relations:

$$\begin{array}{rcl} \dot{x}(t) &=& A_c x(t) + B_c u(t) \\ y(t) &=& C_c x(t) \end{array}, \quad x(t) = \begin{bmatrix} \dot{d}(t) \\ d(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} g(t) \\ 0 \end{bmatrix}, \tag{2}$$

where $x(t)$ is the *state vector*, $u(t)$ the *input vector*, $y(t)$ the *output vector* and:

$$A_c = \begin{bmatrix} -M^{-1}G & -M^{-1}K \\ I & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} M^{-1} \\ 0 \end{bmatrix}, \quad C_c \text{ invertible}, \tag{3}$$

are the resulting model matrices. Note that the experimental measurements are expressed as linear combinations of the displacements $d(t)$ and the velocities $\dot{d}(t)$, defined by the matrix C_c. In the further text we denote $f(x(t), u(t)) = A_c x(t) + B_c u(t)$.

2.1. Model problem

Let us choose as a model problem a system with three masses connected by springs and dampers, as shown in the figure on the left of Table 1, that brings to the following system of equilibrium equations:

$$\begin{cases} M_1\ddot{x}_1(t) = C_1\left(\dot{x}_2(t) - \dot{x}_1(t)\right) + K_1\left(x_2(t) - x_1(t) + \delta x_g\right) + f, \\ M_2\ddot{x}_2(t) = C_2\left(\dot{x}_3(t) - \dot{x}_2(t)\right) + C_1\left(\dot{x}_1(t) - \dot{x}_2(t)\right) \\ \qquad\qquad + K_2\left(x_3(t) - x_2(t) + \delta x_b\right) + K_1\left(x_1(t) - x_2(t) - \delta x_g\right), \\ M_3\ddot{x}_3(t) = C_3\left(-\dot{x}_3(t)\right) + C_2\left(\dot{x}_2(t) - \dot{x}_3(t)\right) \\ \qquad\qquad + K_3\left(-x_3(t) + \delta x_s\right) + K_2\left(x_2(t) - x_3(t) - \delta x_b\right) \quad . \end{cases}$$

In this way we avoid the problem of the model order selection, that would intervene with a system with a lot of degrees-of-freedom or with a continuous system (therefore discretized in space e.g. with the FEM [6]). For a reference on the subject see e.g. [2,10]. Realistic conditions, e.g. dictated by an application in railway engineering, demand for sampling periods $T_{sc} = 0.00001$ and $T_s = 0.001$ and nominal values of the physical parameters as shown in Table 1, that we will take as reference values in our numerical experiments.

Note that this system represents a wide class of models. Indeed, it may be immediately generalized to n masses or the continuum, discretized in space with finite elements [6] or finite differences. The structure of the model would be identical and so the physical meaning of parameters, despite their different number.