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Pricing of options in the singular perturbed stochastic volatility model

Tianmiao Liu, Yoshifumi Muroi *

Graduate School of Economics and Management, Tohoku University, 27-1 Kawauchi, Aoba-Ku, Sendai 980-8576, Japan

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ABSTRACT

The pricing of options in the fast mean-reverting stochastic volatility model using the singular perturbation method has received a considerable amount of attention in the last two decades. However, it is not to easy to estimate the accuracy of the approximation if the payoff function is not smooth or bounded, as is the case for European call options. In this article, we introduce a new novel approach for pricing options in the fast mean-reverting stochastic volatility model. Combinations of Fourier analysis and singular perturbation methods enable us to estimate the accuracy easily. We also show that this method allows us to derive the price of European and Bermudan options in the fast mean-reverting stochastic volatility environment with jumps.

1. Introduction

This paper presents new pricing methods of European options in the fast mean-reverting stochastic volatility model using Fourier analysis and singular perturbation methods. The pricing of various types of options in the fast mean-reverting stochastic volatility environment has been discussed in [1–3] for example. Because this method enables us to derive an asymptotic approximation formula for the price of options in the stochastic volatility models very easily, the pricing of options in the fast mean-reverting stochastic volatility model has received a considerable amount of attention. The pricing of exotic options, for example, has been discussed by Fouque and Han [4], Ilhan et al. [5], and Zhu and Chen [6], among others. Although it is a relatively straightforward task to derive the asymptotical approximation formulas for option prices, it is not easy to estimate its accuracy especially when the payoff function is not smooth. This means that it is not easy to estimate the accuracy of the asymptotic approximation price of European call (put) options because the price of European options creates a singularity at the strike price at the maturity date. See [2,7], for the validity of this method. In order to overcome this difficulty, we use Fourier analysis to obtain the approximation price for European options in the fast mean-reverting stochastic volatility environment. We compute the characteristic functions using partial differential equation (PDE) approach instead of the option price itself. This method enables us to estimate the accuracy of the obtained option prices rigorously and easily because one can assume the smoothness and boundedness to the terminal condition in this PDE. The characteristic function approach is one of the most important approaches for option pricing. Many researchers have been attracted to the tractability of the Fourier transform approach. See [8–11], for example.

The remainder of this paper is organized as follows. The fast mean-reverting stochastic volatility model is introduced in Section 2. In this section, we derive the asymptotic approximation formula for the Fourier transform of the underlying asset price in the fast mean-reverting stochastic volatility model. In Section 3, we show the pricing method of European options in

* Corresponding author. Fax: +81 22 795 6308.
E-mail addresses: dorasensi999@yahoo.co.jp (T. Liu), ymuroi@econ.tohoku.ac.jp (Y. Muroi).

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the fast mean-reverting stochastic volatility model using the Fourier transform approach. We also discuss the validity of this method. In Section 4, we show numerical results using the jump diffusion model with the fast mean-reverting stochastic volatility, followed by concluding remarks in Section 5.

2. Fast mean-reverting stochastic volatility model

We assume that the price of the underlying asset, $S_t = e^{X_t^\epsilon}$, is governed by a stochastic differential equation (SDE) with a stochastic volatility component given by

$$dX_t^\epsilon = \left[r - \frac{1}{2}f^2(Y_t^\epsilon)\right]dt + f(Y_t^\epsilon)\left(\rho dW_t + \sqrt{1-\rho^2}dZ_t\right), \quad X_0^\epsilon = x_0 = \log(s_0) \tag{1}$$

$$dY_t^\epsilon = \left[\frac{1}{\epsilon}(m - Y_t^\epsilon) - \nu\sqrt{\frac{2}{\epsilon}}\Lambda(Y_t^\epsilon)\right]dt + \nu\sqrt{\frac{2}{\epsilon}}dW_t$$

under the risk neutral measure $\mathbb{P}$, where $|\rho| < 1$. The two stochastic processes W_t and Z_t are independent standard Brownian motions, and the constant r (> 0) represents the spot interest rate in this economy. We also assume that ν is a positive constant. The function $\Lambda(y)$ is given by

$$\Lambda(y) = \rho\frac{\mu - r}{f(y)} + \sqrt{1-\rho^2}\gamma(y),$$

where μ is the drift term of the underlying asset price process under the physical measure. We also assume that ϵ is a small positive constant $0 < \epsilon \ll 1$. We also impose the following assumptions upon the functions $f(\cdot)$ and $\gamma(\cdot)$:

Assumption 2.1. We impose the following assumptions upon $f(\cdot)$ and $\gamma(\cdot)$,

(1) The volatility function $f(\cdot)$ is a positive, bounded, and bounded away from 0. In other words, we assume the relationship $0 < \underline{c} < f(y) < \bar{c} < \infty$ for all y.
(2) The market price of the volatility risk given by $\gamma(y)$ is a bounded function, i.e. $|\gamma(y)| < l < \infty$.

In this study, we compute the characteristic function of the log-price of the underlying asset at the maturity date T using the singular perturbation methods introduced by Fouque et al. [1]. The conditional characteristic function of the random variable X_T^ϵ is denoted by Ψ_{X^ϵ}:

$$\Psi_{X^\epsilon}(t, x, y, \theta) = \mathbb{E}[e^{i\theta X_T^\epsilon}|X_t^\epsilon = x, Y_t^\epsilon = y].$$

The Feynman and Kac theorem leads to a PDE for Ψ_{X^ϵ} given by

$$\mathscr{L}^\epsilon\Psi_{X^\epsilon} = 0, \qquad \Psi_{X^\epsilon}(T, x, y, \theta) = e^{i\theta x} \tag{2}$$

where the new operator $\mathscr{L}^\epsilon$ is given by

$$\mathscr{L}^\epsilon = \frac{1}{\epsilon}\mathscr{L}_0 + \frac{1}{\sqrt{\epsilon}}\mathscr{L}_1 + \mathscr{L}_2,$$

$$\mathscr{L}_0 = \nu^2\frac{\partial^2}{\partial y^2} + (m - y)\frac{\partial}{\partial y},$$

$$\mathscr{L}_1 = \sqrt{2}\rho\nu f(y)\frac{\partial^2}{\partial x\partial y} - \sqrt{2}\nu\Lambda(y)\frac{\partial}{\partial y},$$

$$\mathscr{L}_2 = \frac{\partial}{\partial t} + \frac{1}{2}f^2(y)\left[\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial x}\right] + r\frac{\partial}{\partial x}.$$

Note that the terminal condition for the PDE (2) indicates that the smoothness of Ψ_{X^ϵ} at T. We assume that there is a formal asymptotic expansion of Ψ_{X^ϵ} on the parameter ϵ as

$$\Psi_{X^\epsilon} = \Psi_0 + \sqrt{\epsilon}\Psi_1 + \epsilon\Psi_2 + \cdots.$$

Inserting this expansion into the PDE in (2), we obtain an equation for the $O(\epsilon^{-1})$-order term given by

$$\mathscr{L}_0\Psi_0 = 0.$$

Note that the characteristic function is a bounded function and Ψ_0 does not depend on the variable y, *i.e.* $\Psi_0 = \Psi_0(t, x, \theta)$. As the $O(\epsilon^{-1})$-order term, we equate the $O(\epsilon^{-1/2})$-order term to 0, and obtain

$$\mathscr{L}_0\Psi_1 + \mathscr{L}_1\Psi_0 = 0.$$