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A deterministic model for the distribution of the stopping time in a stochastic equation and its numerical solution

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Abstract

In this work, we consider a stochastic differential equation that generalizes the well known Paris' equation from fracture of materials. The model describes the propagation of cracks on solids, and it includes a deterministic summand and a stochastic component in terms of a Brownian motion. The use of Itô's stochastic integral gives an equivalent stochastic integral equation that is further generalized here. We note that the probability distribution of the stopping time of the general model satisfies a deterministic diffusion-advection partial differential equation for which the solution is known only in a reduced number of particular cases. Motivated by these analytical results, we develop a fast finite-difference method to approximate the distribution of the stopping time. The method is an explicit exponential-like technique that preserves the main features of a probability distribution, namely, the non-negativity, the boundedness from above by 1 as well as the spatial monotonicity. Moreover, the method is a monotone technique that is also capable of preserving the temporal monotonicity of the approximations. These properties of the proposed methodology are thoroughly established in the present manuscript. A continuity condition of the numerical solutions in terms of the initial conditions and the temporal computational parameter is established also, together with a limiting property of the methodology when the free parameter tends to infinity. For comparison purposes, we are providing an implicit and stable discretization of the mathematical model which has a second order of convergence but for which conditions that guarantee the positivity, the boundedness and the monotonicity of approximations are not available. The numerical simulations obtained with implementations of our techniques show that the explicit method is an efficient scheme that preserves the characteristics of interest (non-negativity, boundedness from above by 1 and monotonicity), and that the numerical approximations are in good agreement with the known exact solutions.

Keywords: stochastic differential equations, nonlinear partial differential equations, Paris' equation, probability distribution of hitting time, probability-based numerical method, generalized exponential technique
2010 MSC: 65N06, 65C20, 35C05, 35K20

1. Introduction

Since the introduction of the well known Paris' law of fracture mechanics [1], various generalizations of this model have been reported in the literature. Paris' law is a simple ordinary differential equation that describes the propagation of cracks on solid materials, and it has found many interesting applications in the natural sciences and engineering [2, 3, 4, 5, 6]. The model is a power-law obtained from empirical observations, and its generalizations have been of great interest among scientists in physics, structural mechanics, mathematics and statistics [7, 8, 9]. Indeed, the incorporation of stochastic components has helped to account for the typical variability on the dynamics of cracks on solids [10, 11]. In that sense, the stochastic Paris' equations are more realistic models that have found a wide variety of practical applications. As expected, this direction of mathematical and stochastic research is a very active avenue of investigation nowadays. Many models have been proposed and their properties have been established analytically.

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