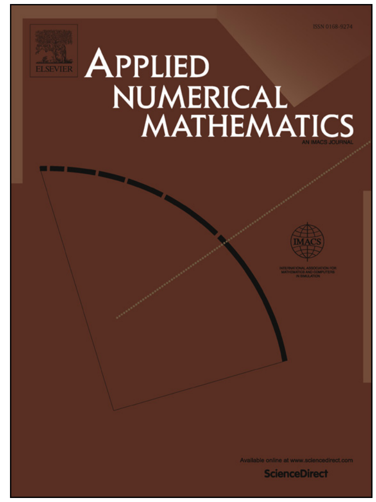
Accepted Manuscript



Numerical simulation of Bloch equations for dynamic magnetic resonance imaging

Arijit Hazra, Gert Lube, Hans-Georg Raumer

PII: S0168-9274(17)30201-5
DOI: https://doi.org/10.1016/j.apnum.2017.09.007
Reference: APNUM 3257

To appear in: *Applied Numerical Mathematics*

Received date: 15 March 2017
Revised date: 11 September 2017
Accepted date: 19 September 2017

Numerical Simulation of Bloch Equations for Dynamic Magnetic Resonance Imaging

Arijit Hazra*, Gert Lube*, Hans-Georg Raumer

Institute for Numerical and Applied Mathematics, Georg-August-University Göttingen, D-37083, Germany

Abstract

Magnetic Resonance Imaging (MRI) is a widely applied non-invasive imaging modality based on non-ionizing radiation which gives excellent images and soft tissue contrast of living tissues. We consider the modified Bloch problem as a model of MRI for flowing spins in an incompressible flow field. After establishing the well-posedness of the corresponding evolution problem, we analyze its spatial semi-discretization using discontinuous Galerkin methods. The high frequency time evolution requires a proper explicit and adaptive temporal discretization. The applicability of the approach is shown for basic examples.

Keywords: Magnetic resonance imaging, Bloch model, FLASH-technology, flowing spins, incompressible medium, discontinuous Galerkin method, explicit Runge-Kutta methods

1. Bloch Model for Magnetic Resonance Imaging

Magnetic Resonance Imaging (MRI) is a non-invasive imaging modality based on non-ionizing radiation [1]. It gives excellent images and soft tissue contrast of living tissues. During the experiment, the object to be studied is placed in a static magnetic field of high strength B_0. This induces a macroscopic nuclear magnetization $\mathbf{M}_0$ in the direction of magnetic field, known as the equilibrium magnetization. The direction of equilibrium magnetization is called the longitudinal direction, generally denoted by the z-axis as in Figure 1.

In order to get a response from the object, the equilibrium magnetization is perturbed by applying a short radio-frequency (RF) pulse $\mathbf{B}_1$ in the transverse plane (denoted by the xy-plane in Figure 1) with the excitation carrier frequency of the RF pulse equal to the Larmor frequency of protons. As a result, the equilibrium magnetization is flipped from its initial position towards the transverse plane. The perturbation of equilibrium

*Corresponding authors

Email addresses: `arijit.hazra@stud.uni-goettingen.de` (Arijit Hazra), `lube@math.uni-goettingen.de` (Gert Lube), `hansgeorg.raumer@stud.uni-goettingen.de` (Hans-Georg Raumer)