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Characterising inflations of monotone grid classes of permutations

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ABSTRACT

We characterise those permutation classes whose simple permutations are monotone griddable. This characterisation is obtained by identifying a set of nine substructures, at least one of which must occur in any simple permutation containing a long sum of 21s.

1. Introduction

A common route to understanding the structure of a permutation class (and hence, e.g. complete its enumeration) is via its simple permutations, as their structure can be considerably easier to characterise than the entire class. Albert, Atkinson, Homberger and Pantone [4] introduced the notion of *deflatability* to study this phenomenon: that is, the property that the simples in a given permutation class $\mathcal{C}$ actually belong to a proper subclass $\mathcal{D} \subsetneq \mathcal{C}$. See also Vatter's recent survey [14].

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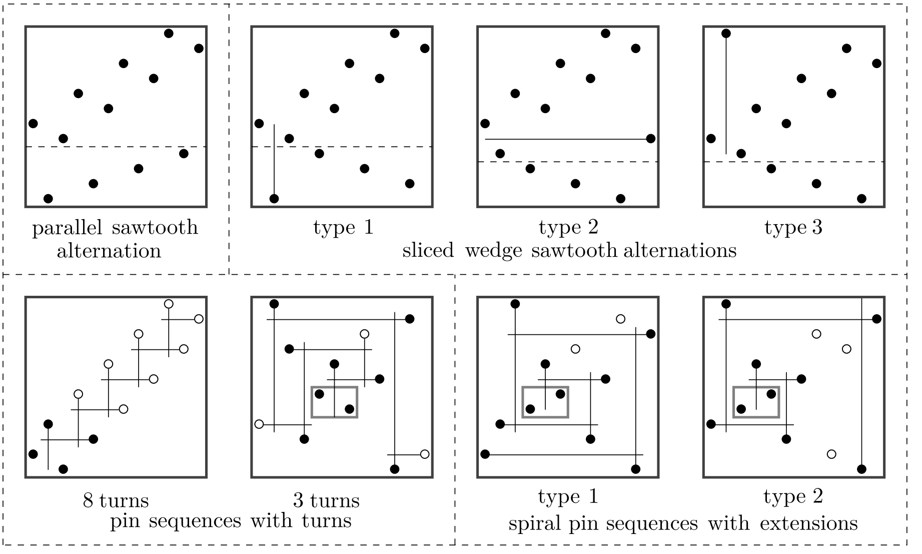

Fig. 1. Examples of the permutations characterising the griddability of simples in Theorem 1.1.

One general case of deflatability is where the set of simple permutations of a class is finite. Such classes are well-quasi-ordered, finitely based, and have algebraic generating functions [1], and via a Ramsey-type result for simple permutations [9], it is decidable when a permutation class has this property [10].

In this paper, we look beyond classes with finitely many simples to those whose simples are 'monotone griddable', and prove the following characterisation. We postpone formal definitions until later, but see Fig. 1 for examples of the structures mentioned.

Theorem 1.1. *The simple permutations in a class $\mathcal{C}$ are monotone griddable if and only if $\mathcal{C}$ does not contain the following structures, or their symmetries:*

- *arbitrarily long parallel sawtooth alternations,*
- *arbitrarily long sliced wedge sawtooth alternations,*
- *proper pin sequences with arbitrarily many turns, and*
- *spiral proper pin sequences with arbitrarily many extensions.*

In general, classes whose simple permutations are monotone griddable do not immediately possess the range of properties that classes with only finitely many simples do. Indeed, few general properties are known even for classes that are themselves wholly monotone griddable, but this has not diminished the efficacy of the following characterisation for the structural understanding and enumeration of many classes (see, for example [3]).