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Minimal asymmetric graphs

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ABSTRACT

Confirming a conjecture of Nešetřil, we show that up to isomorphism there are only finitely many finite minimal asymmetric undirected graphs. In fact, there are exactly 18 such graphs. We also show that these graphs are exactly the finite minimal involution-free graphs.

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1. Introduction

A graph is *asymmetric* if it does not have a nontrivial automorphism. In this paper, we are interested in asymmetric graphs that are as small as possible. An undirected graph G on at least two vertices is *minimal asymmetric* if G is asymmetric and no proper induced subgraph of G on at least two vertices is asymmetric.

In 1988 Nešetřil conjectured at an Oberwolfach Seminar that there exists only a finite number of finite minimal asymmetric graphs, see [16]. Since then Nešetřil and Sabidussi have made significant progress on the conjecture. They showed that there are exactly nine minimal asymmetric graphs containing P_5, the path of length 4, as an induced subgraph [11,14,15] and identified 18 minimal asymmetric graphs in total. However, the conjecture has remained open over the years and has been mentioned in various other publications

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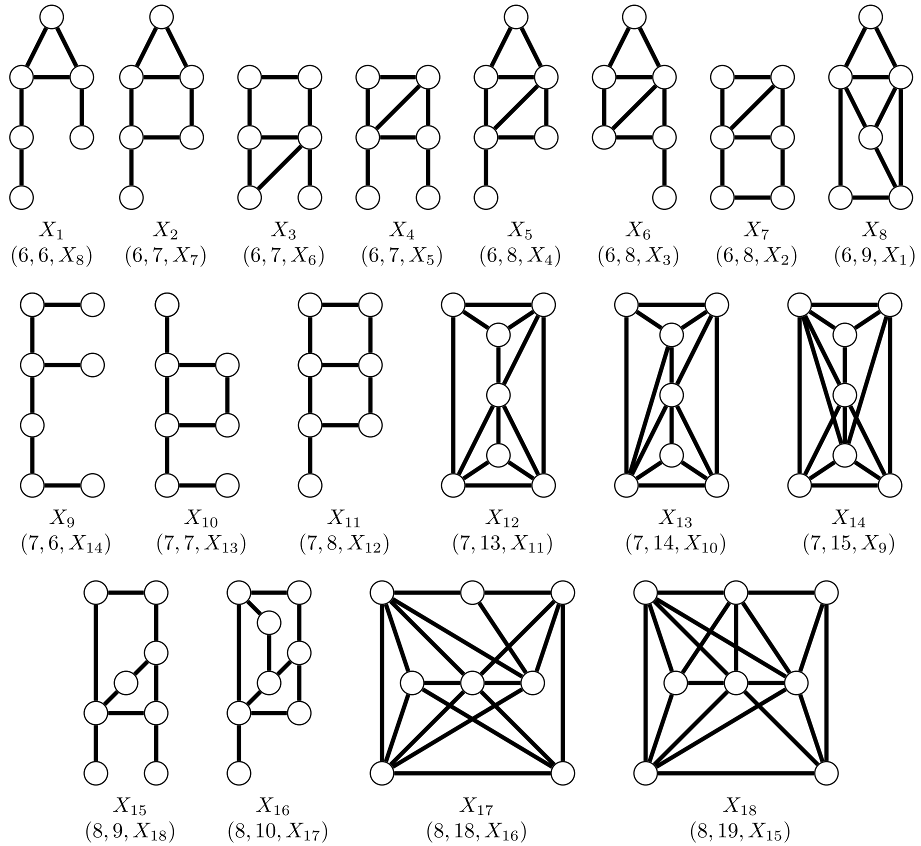

Fig. 1. The 18 minimal asymmetric graphs. These are also the minimal involution-free graphs. For each graph the triple $(n, m, \text{co-}G)$, describes the number of vertices, edges and the name of the complement graph, respectively. The graphs are ordered first by number of vertices and second by number of edges.

[2,8,12,13]. Coincidentally, Nešetřil mentioned the open conjecture as recently as 2016 at an Oberwolfach Seminar. We now confirm the conjecture.

Theorem 1. *There are exactly 18 finite minimal asymmetric undirected graphs up to isomorphism. These are the 18 graphs depicted in Fig. 1.*

A classic result of Erdős and Rényi [5] says that as n tends to infinity the fraction of graphs on n vertices that are asymmetric tends to 1, and it is not difficult to see that every finite graph embeds into a finite asymmetric graph, so Theorem 1 may come as surprise.

In their papers, Nešetřil and Sabidussi found a close connection between minimal asymmetric graphs and minimal involution-free graphs. A graph is *involution-free* if it does not have an automorphism of order 2. An undirected graph G on at least 2 vertices is *minimal involution-free* if G is involution-free and no proper induced subgraph of G on at least 2 vertices is involution-free. Nešetřil and Sabidussi conjectured that the set

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