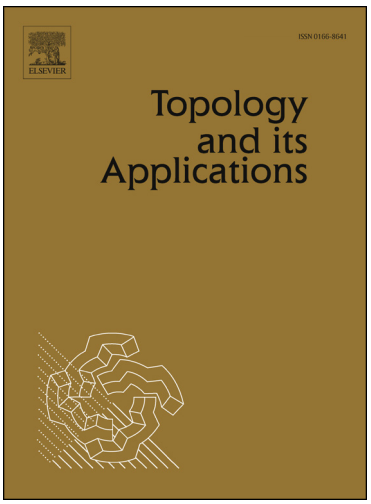

Three-space properties in paratopological groups

Manuel Fernández, Iván Sánchez

PII: S0166-8641(16)30301-7
DOI: http://dx.doi.org/10.1016/j.topol.2016.11.011
Reference: TOPOL 5954

To appear in: *Topology and its Applications*

Received date: 11 August 2016
Revised date: 12 November 2016
Accepted date: 13 November 2016

Please cite this article in press as: M. Fernández, I. Sánchez, Three-space properties in paratopological groups, *Topol. Appl.* (2016), http://dx.doi.org/10.1016/j.topol.2016.11.011

Three-space properties in paratopological groups

Manuel Fernández

Academia de Matemáticas

Universidad Autónoma de la Ciudad de México

Prolongación San Isidro 151, Col. San Lorenzo Tezonco, Del. Iztapalapa, C.P. 09790

Mexico, D.F., Mexico

manuel.fernandezvillanueva@uacm.edu.mx, mafevil5@gmail.com

Iván Sánchez

Departamento de Matemáticas

Universidad Autónoma metropolitana

Av. San Rafael Atlixco 186, Col. Vicentina

Del. Iztapalapa, C.P. 09340, Mexico city

isr.uami@gmail.com

November 11

Abstract

We show that neither first-countable nor second-countable are three-space properties in the class of paratopological groups: we present a countable regular paratopological Abelian group H which contains a closed discrete subgroup F such that H/F is topologically isomorphic to the rational numbers with the Sorgenfrey topology and H is not first-countable. Also, we prove that if H is an invariant topological subgroup of a paratopological group G such that H is second-countable and G/H has countable network, then G has countable network as well (this answers a question posed in [12]). Hence if H is an invariant topological subgroup of a first-countable paratopological group G such that H and G/H are second-countable, then so is G.

MSC (2010): Primary 54H11, 22A05, Secondary 54B15, 54D30, 54E35

Keywords: Three-space property; Paratopological group; First-countable; Second-countable; Countable network

1 Introduction

A (*semitopological*) *paratopological group* G is a group G with a topology which make multiplication in G (separately) continuous. If, in addition, the inversion in G is continuous, the group G is