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Characterizing model-theoretic dividing lines via collapse of generalized indiscernibles

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ABSTRACT

We use the notion of collapse of generalized indiscernible sequences to classify various model theoretic dividing lines. In particular, we use collapse of n-multi-order indiscernibles to characterize op-dimension n; collapse of function-space indiscernibles (i.e. parameterized equivalence relations) to characterize rosy theories; and finally, convex equivalence relation indiscernibles to characterize NTP2 theories.

1. Introduction

In model theory, and in S. Shelah's classification theory in particular, one central program is the search for robust dividing lines among complete theories – dividing lines between intelligibility and non-structure. For a dividing line to be sufficiently interesting, one often desires that both sides of the line have interesting mathematical content. Moreover, if such a dividing line has multiple characterizations coming from seemingly different contexts, this lends credence to the notion that the line is actually substantial. The exemplar of this in model theory is the notion of stability, first introduced by S. Shelah [9]. Both stable and unstable theories are inherently interesting, and stability enjoys many different characterizations, from cardinalities of Stone spaces, to coding orders, to collapse of indiscernible sequences to indiscernible sets. Another example of such a robust dividing line is NIP; like stability, there are important structures with theories on both sides of the line, and the model theory on both sides can be very rich.

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In this paper, we focus on the notion of collapsing generalized indiscernibles as a means of characterizing/defining various model-theoretic dividing lines that are already well-established in the literature. It can be said that this work started with Shelah's original characterization of stable theories, Theorem II.2.13 of [9]. There he shows that a theory is stable if and only if every indiscernible sequence is an indiscernible set (i.e., the order of the sequence "does not matter" or is invisible to models of the theory). In a similar fashion, it can be shown that other well-known model-theoretic dividing lines can be characterized similarly by such "collapse" statements, which we formalize here. In order to carry this out, we must expand our definition of "indiscernibility." A quintessential example of this phenomenon beyond stability is the third author's characterization of NIP theories by ordered-graph indiscernibles. In [8], she shows that a theory is NIP if and only if any indiscernible "picture" of the generic ordered graph is actually indiscernible *without* the graph structure (i.e., it collapses to order in the sense that the graph relation is invisible to models of the theory).

The project in this paper was also alluded to in the work of the first and second authors on op-dimension [3], and this will be discussed further in Section 3 below. Generalized indiscernibles were first introduced by Shelah (see Section VII.2 of [9]), where they were used in the context of tree-indexed indiscernibles in the hope of understanding the tree property (see, for example, Theorem III.7.11 of [9]), but we propose a slightly different formulation which seems to simplify some aspects of Section 4. For more on tree indiscernibles in particular, see [5,6].

Outline of the paper. In Section 2, we introduce all the relevant notation and notions for generalized indiscernibility, the Ramsey Property, and the Modeling Property. In the remaining sections, we will apply this to specific cases, exhibiting the "collapse" characterizations of various dividing lines. In Section 3, we begin with the example of n-multi-order indiscernibles, first considered in [3], and show that collapse down to n orders characterizes op-dimension n (Theorem 3.4 below). In Section 4, we consider function-space indiscernibles, and show that collapse down to two indiscernible sequences characterizes rosiness (Theorem 4.7 below). In Section 5, we consider convex equivalence relation indiscernibles, showing that NTP2 is equivalent to a dichotomy between collapsing down to an indiscernible sequence or having dividing over the indiscernible parameters be dynamic in some way ("dividing across I implies dividing vertically across I") (Theorem 5.9 below).

2. Theories of (generalized) indiscernibles

In this section, we review the definitions associated with structural Ramsey theory: Fraïssé-like theories, the Ramsey Property, and the Modeling Property, and theories of indiscernibles. In Subsection 2.4, we formalize the notion of "collapse of indiscernibles" as it will be used in this article.

2.1. Notation and conventions for finite structures

Definition 2.1 *(Notation for finite structures).* We use plain upper-case Roman letters – like A, B, C and so forth – to denote finite structures, and upper-case calligraphic letters – $\mathcal{A}, \mathcal{B}, \mathcal{M}, \mathcal{N}$ and so forth – usually denote infinite structures. (There is some ambiguity in that the universe of a structure $\mathcal{A}$ is usually denoted A, but we will clarify as necessary. In fact, except in this section, we have little need for abstractly presented finite structures.)

We write $\mathrm{Emb}(A, \mathcal{M})$, $\mathrm{Emb}(A, B)$, $\mathrm{Emb}(\mathcal{M}, \mathcal{N})$ for the sets of all embeddings $A \to \mathcal{M}$, $A \to B$ and $\mathcal{M} \to \mathcal{N}$, respectively. For an embedding u in one of these sets, uA (or $u\mathcal{M}$) is the substructure of the codomain induced on the image of u.

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