Journal of Applied Logic ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

Factored performance functions and decision making in continuous time Bayesian networks

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Continuous time Bayesian network
Performance function
Synergy
Multi-objective optimization

ABSTRACT

The continuous time Bayesian network (CTBN) is a probabilistic graphical model that enables reasoning about complex, interdependent, and continuous-time subsystems. The model uses nodes to denote subsystems and arcs to denote conditional dependence. This dependence manifests in how the dynamics of a subsystem changes based on the current states of its parents in the network. While the original CTBN definition allows users to specify the dynamics of how the system evolves, users might also want to place value expressions over the dynamics of the model in the form of performance functions. We formalize these performance functions for the CTBN and show how they can be factored in the same way as the network, allowing what we argue is a more intuitive and explicit representation. For cases in which a performance function must involve multiple nodes, we show how to augment the structure of the CTBN to account for the performance interaction while maintaining the factorization of a single performance function for each node. We introduce the notion of optimization for CTBNs, and show how a family of performance functions can be used as the evaluation criteria for a multi-objective optimization procedure.

1. Introduction

Many problems in artificial intelligence require reasoning about complex systems. One important and difficult type of system is one that changes through time. Temporal modeling and reasoning present additional challenges in representing the system's dynamics while efficiently and accurately inferring the system's behavior through time. Continuous time Bayesian networks (CTBNs) were introduced by Nodelman, Shelton, and Koller [14] as a temporal model capable of representing and reasoning about finite- and discrete-state systems without committing to a uniform discretization of time, as found with dynamic Bayesian networks [11].

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http://dx.doi.org/10.1016/j.jal.2016.11.030
1570-8683/© 2016 Published by Elsevier B.V.

Since then, the CTBN has found use in a wide variety of applications. For example, they have been used for inferring users' presence, activity, and availability over time [13]; robot monitoring [12]; modeling server farm failures [9]; modeling social network dynamics [5]; modeling sensor networks [17]; building intrusion detection systems [21–23]; predicting the trajectory of moving objects [16]; diagnosing cardiogenic heart failure and anticipating its likely evolution [7,8], and reasoning about complex reliability models [1,20].

While most of these applications are concerned with the most probable state of the system at certain times, we can see the advantage of moving beyond this and also estimating user-specified *values* on how the system behaves. Users may have complex valuations on how and when failures occur, intrusions are detected, diagnoses are made, etc. In this extended version of our conference paper [18], we introduce families of factored performance functions into the CTBN to support this idea. We then formalize the concept of optimization in CTBNs, and show how families of performance functions can be used as the evaluation criteria for a multi-objective optimization problem. We demonstrate the use of factored performance functions with three networks. In the first network, we show how factoring the performance function can increase the efficiency of inference without loss in accuracy. The next two networks demonstrate how families of factored performance functions can be used for CTBN multi-objective optimization.

2. Background

Before we describe these factored performance functions, we must formally define the BN and then the CTBN. We then discuss the representation of the instantiations of the network, and review prior work in CTBN inference that allows a user to estimate arbitrary functions over the behavior of the network.

2.1. Bayesian networks

Bayesian networks are probabilistic models corresponding to joint probability distributions that utilize conditional dependencies among random variables. Bayesian networks are used in a wide variety of domains, such as image processing, search, information retrieval, diagnostics, and many others. A Bayesian network uses observations, or evidence, and previously determined conditional probabilities to give the probability of a certain state.

Definition 1 *(Bayesian network).* A Bayesian network $\mathcal{B}$ is a directed, acyclic graph whose vertices correspond to random variables of a distribution, and the edges correspond to conditional dependencies between random variables. Each vertex has an associated conditional probability distribution, denoted $P(X_i|\mathrm{Pa}(X_i))$, where $\mathrm{Pa}(X_i)$ are the direct predecessors (also called "parents") of vertex X_i.

Bayesian networks are a way of representing joint probability distributions in a more compact way by using conditional dependencies among the random variables. Instead of needing to enumerate the entire joint probability distribution we can just use the product rule from probability to get the following:

$$P(X_1, ..., X_n) = P(X_1) \prod_{i=2}^{n} P(X_i|X_1, ..., X_{i-1})$$

Bayesian networks are able to exploit conditional independence, which is represented in the directed acyclic graph $\mathcal{G}$, to reduce the model's complexity and yield the following:

$$P(X_1, ..., X_n) = \prod_{i=1}^{n} P(X_i|\mathrm{Pa}(X_i)).$$

Please cite this article in press as: L. Sturlaugson et al., Factored performance functions and decision making in continuous time Bayesian networks, J. Appl. Log. (2016), http://dx.doi.org/10.1016/j.jal.2016.11.030