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On the existence of free models in fuzzy universal Horn classes

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ABSTRACT

This paper is a contribution to the study of the universal Horn fragment of predicate fuzzy logics, focusing on some relevant notions in logic programming. We introduce the notion of *term structure associated to a set of formulas* in the fuzzy context and we show the existence of free models in fuzzy universal Horn classes. We prove that every equality-free consistent universal Horn fuzzy theory has a Herbrand model.
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1. Introduction

Since their introduction in [28], Horn clauses have shown to have good logic properties and have proven to be of importance for many disciplines, ranging from logic programming, abstract specification of data structures and relational data bases, to abstract algebra and model theory. However, the analysis of Horn clauses has been mainly restricted to the sphere of classical logic. For a good exposition of the most relevant results concerning Horn clauses in classical logic we refer to [24], and to [26] for a good study of their importance in computer science.

The interest in continuous t-norm based logics since its systematization by Hájek [23] and the subsequent study of core fuzzy logics [9] invite to a systematic development of a model theory of these logics (and of algebraizable non-classical logics in general). Cintula and Hájek raised the open question of characterizing theories of Horn clauses in predicate fuzzy logics [9]. Our first motivation to study the Horn fragment of predicate fuzzy logics was to solve this open problem, the present article is a first contribution towards its solution.

Some authors have contributed to the study of Horn clauses over fuzzy logic. In [6,5,4,2,3,31] Bělohlávek and Vychodil study fuzzy equalities, they work with theories that consist of formulas that are implications between identities with premises weighted by truth degrees. They adopt Pavelka style: theories are fuzzy sets

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of formulas and they consider degrees of provability of formulas from theories. Their basic structure of truth degrees is a complete residuated lattice. The authors derive a Pavelka-style completeness theorem (degree of provability equals degree of truth) from which they get some particular cases by imposing restrictions on the formulas under consideration. As a particular case, they obtain completeness of fuzzy equational logic. In different articles they study the main logical properties of varieties of algebras with fuzzy equalities. Taking a different approach, in a series of papers [21,20,19], Gerla proposes to base fuzzy control on fuzzy logic programming, and observes that the class of fuzzy Herbrand interpretations gives a semantics for fuzzy programs. Gerla works with a complete, completely distributive, lattice of truth-values. For a reference on fuzzy logic programming see [30,17].

Several definitions of Horn clause have been proposed in the literature of fuzzy logics, but there is not a canonical one yet. Cintula and Hájek affirm that the elegant approach of [2] is not the only possible one. In [15], Dubois and Prade discuss different possibilities of defining *fuzzy rules* and they show how these different semantics can be captured in the framework of fuzzy set theory and possibility theory. Following all these works, our contribution is a first step towards a systematic model-theoretic account of Horn clauses in the framework introduced by Hájek in [23]. We introduce a basic definition of Horn clause over the predicate fuzzy logic MTL$\forall^m$ that extends the classical one in a natural way. In future work we will explore different generalizations of our definitions for expanded languages. Our approach differs from the one of Bělohlávek and Vychodil because we do not restrict to fuzzy equalities. Another difference is that, unlike these authors and Gerla, our structures are not necessarily over the same complete algebra, because we work in the general semantics of [23].

In the present work we have focused on the study of *free models of Horn clauses*. Free structures have a relevant role in classical model theory and logic programming. Admitting free structures makes reasonable the concepts of *closed-word assumption* for databases and *negation as failure* for logic programming. These structures allow also a procedural interpretation for logic programs (for a reference see [26]). Free structures of a given class are minimal from an algebraic point of view, in the sense that there is a unique homomorphism from these structures to any other structure in the class. The free structures introduced here are *term structures*, structures whose domains consist of terms or equivalence classes of terms of the language. In classical logic, term structures have been used to prove the satisfiability of a set of consistent sentences, see for instance [16, Ch.5]. Notorious examples of term structures are Herbrand models, they play an important function in the foundations of logic programming. Several authors have been studied Herbrand models in the fuzzy context (for a reference see [19,30,17]), providing theoretical background for different classes of fuzzy expert systems. For a general reference on Herbrand Theorems for substructural logics we refer to [7].

The present paper is an extension of the work presented in the 18th International Conference of the Catalan Association for Artificial Intelligence (CCIA 2015) [11]. Our main original contributions are the following:

- Introduction of the notion of term structure associated to a theory over predicate fuzzy logics. If the theory consists of universal Horn formulas, we show that the associated term structure is a model of the theory (Theorem 2).
- Existence of free models in fuzzy universal Horn classes of structures. In the case that the language has an equality symbol $\approx$ interpreted as a similarity, we prove the existence of models which are free in the class of reduced models of the theory (Theorem 1). In the case that the language has the crisp identity, the class has free models in the usual sense.
- Consistent universal Horn theories over predicate fuzzy logics (that contains only the truth-constants $\overline{1}$ and $\overline{0}$) have classical models (Corollary 1).
- Introduction of Herbrand structures. We prove that every equality-free consistent universal Horn theory over predicate fuzzy logics have a Herbrand model (Corollary 2).

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