

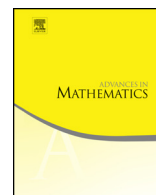


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## Copies of the random graph

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## ABSTRACT

Let  $\langle R, \sim \rangle$  be the Rado graph,  $\text{Emb}(R)$  the monoid of its self-embeddings,  $\mathbb{P}(R) = \{f(R) : f \in \text{Emb}(R)\}$  the set of copies of  $R$  contained in  $R$ , and  $\mathcal{I}_R$  the ideal of subsets of  $R$  which do not contain a copy of  $R$ . We consider the poset  $\langle \mathbb{P}(R), \subset \rangle$ , the algebra  $P(R)/\mathcal{I}_R$ , and the inverse of the right Green's preorder on  $\text{Emb}(R)$ , and show that these preorders are forcing equivalent to a two step iteration of the form  $\mathbb{P} * \pi$ , where the poset  $\mathbb{P}$  is similar to the Sacks perfect set forcing: adds a generic real, has the  $\aleph_0$ -covering property and, hence, preserves  $\omega_1$ , has the Sacks property and does not produce splitting reals, while  $\pi$  codes an  $\omega$ -distributive forcing. Consequently, the Boolean completions of these four posets are isomorphic and the same holds for each countable graph containing a copy of the Rado graph.

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### 1. Introduction

In this paper we continue the investigation of the partial orderings of the form  $\langle \mathbb{P}(\mathbb{X}), \subset \rangle$ , where  $\mathbb{X}$  is an ultrahomogeneous relational structure and  $\mathbb{P}(\mathbb{X})$  the set of domains of substructures of  $\mathbb{X}$  isomorphic to  $\mathbb{X}$ . In particular, if  $\mathbb{X} = \langle X, \rho \rangle$  is a binary structure (that is  $\rho \subset X \times X$ ), then  $\mathbb{P}(\mathbb{X}) = \{A \subset X : \langle A, \rho_A \rangle \cong \langle X, \rho \rangle\}$ , where  $\rho_A = \rho \cap (A \times A)$ . In the sequel, in order to simplify notation, instead of  $\langle \mathbb{P}(\mathbb{X}), \subset \rangle$  we will write  $\mathbb{P}(\mathbb{X})$  whenever the context admits.

This investigation is related to a coarse classification of relational structures. Namely, the conditions  $\mathbb{P}(\mathbb{X}) = \mathbb{P}(\mathbb{Y})$ ,  $\mathbb{P}(\mathbb{X}) \cong \mathbb{P}(\mathbb{Y})$ ,  $\text{sq } \mathbb{P}(\mathbb{X}) \cong \text{sq } \mathbb{P}(\mathbb{Y})$  and  $\text{ro sq } \mathbb{P}(\mathbb{X}) \cong \text{ro sq } \mathbb{P}(\mathbb{Y})$  (where  $\text{sq } \mathbb{P}$  denotes the separative quotient of a partial order  $\mathbb{P}$  and  $\text{ro sq } \mathbb{P}$  its Boolean completion) define different equivalence relations (“similarities”) on the class of relational structures and their interplay with the similarities defined by the conditions  $\mathbb{X} = \mathbb{Y}$ ,  $\mathbb{X} \cong \mathbb{Y}$  and  $\mathbb{X} \rightleftarrows \mathbb{Y}$  (equimorphism) was considered in [11]. It turns out that the similarity defined by the condition  $\text{ro sq } \mathbb{P}(\mathbb{X}) \cong \text{ro sq } \mathbb{P}(\mathbb{Y})$  is implied by all the similarities listed above and, thus, provides the coarsest among the mentioned classifications of relational structures. Since the posets of copies are always homogeneous, the condition  $\text{ro sq } \mathbb{P}(\mathbb{X}) \cong \text{ro sq } \mathbb{P}(\mathbb{Y})$  is equivalent to the forcing equivalence of the posets  $\mathbb{P}(\mathbb{X})$  and  $\mathbb{P}(\mathbb{Y})$  (we will write  $\mathbb{P}(\mathbb{X}) \equiv \mathbb{P}(\mathbb{Y})$ ) and, for convenience, we will exploit this fact using the tools of set-theoretic forcing in our proofs.

This paper can also be regarded as a part of the investigation of the quotient algebras of the form  $P(\omega)/\mathcal{I}$ , where  $\mathcal{I}$  is an ideal on  $\omega$ . Namely, by [8], if  $\mathbb{X}$  is a countable indivisible<sup>1</sup> structure with domain  $\omega$ , then the collection  $\mathcal{I}_{\mathbb{X}}$  of subsets of  $\omega$  which do not contain a copy of  $\mathbb{X}$  is either the ideal of finite sets or a co-analytic tall ideal and the poset  $\text{sq } \mathbb{P}(\mathbb{X})$  is isomorphic to a dense subset of  $(P(\omega)/\mathcal{I}_{\mathbb{X}})^+$ , which implies  $\text{ro sq } \mathbb{P}(\mathbb{X}) \cong \text{ro}(P(\omega)/\mathcal{I}_{\mathbb{X}})^+$ . So, since the structure considered in this paper, the Rado graph,  $\langle R, \sim \rangle$ , is indivisible, our results can be regarded as statements concerning the forcing related properties of the corresponding quotient algebra. Namely, if we call a graph *scattered* if it does not contain a copy of the Rado graph, and if  $\mathcal{I}_R$  denotes the ideal of scattered subgraphs of  $R$ , then

$$\text{ro sq } \mathbb{P}(R) = \text{ro}((P(R)/\mathcal{I}_R)^+).$$

As a consequence of the main result of [14] we have the following statement describing the forcing related properties of the poset of copies of the rational line,  $\mathbb{Q}$ , and the corresponding quotient  $P(\mathbb{Q})/\text{Scatt}$ , where  $\text{Scatt}$  denotes the ideal of scattered suborders of  $\mathbb{Q}$ . Namely, if  $\mathbb{S}$  denotes the Sacks perfect set forcing and  $\text{sh}(\mathbb{S})$  the size of the continuum in the Sacks extension, then we have

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<sup>1</sup> A relational structure  $\mathbb{X} = \langle X, \rho \rangle$  is *indivisible* (resp. *strongly indivisible*) iff for each partition of its domain  $X$  into two pieces one of them contains a copy of  $\mathbb{X}$  (resp. one of them is a copy of  $\mathbb{X}$ ). It is easy to see that the rational line,  $\mathbb{Q}$ , is an indivisible structure which is not strongly indivisible.

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