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On subspace-diskcyclicity

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Abstract. In this paper, we define and study subspace-diskcyclic operators. We show that subspace-diskcyclicity does not imply diskcyclicity. We establish a subspace-diskcyclic criterion and use it to find a subspace-diskcyclic operator that is not subspace-hypercyclic for any subspaces. Also, we show that the inverses of invertible subspace-diskcyclic operators do not need to be subspace-diskcyclic for any subspaces. Finally, we prove that every finite-dimensional Banach space over the complex field supports a subspace-diskcyclic operator.

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1. Introduction

A bounded linear operator T on a separable Banach space $\mathcal{X}$ is hypercyclic if there is a vector $x \in \mathcal{X}$ such that its orbit $Orb(T, x) = \{T^n x : n \geq 0\}$ is dense in $\mathcal{X}$; such a vector x is called hypercyclic for T. The first example of a hypercyclic operator on a Banach space was constructed by Rolewicz in 1969 [11]. He showed that if B is the backward shift on $\ell^p(\mathbb{N})$ then λB is hypercyclic if and only if $|\lambda| > 1$.

The study of the scaled orbit and disk orbit is motivated by the Rolewicz example [11]. In 1974, Hilden and Wallen [7] defined the notion of supercyclicity. An operator T is called supercyclic if there is a vector x such that its scaled orbit $\mathbb{C}Orb(T, x)$ is dense in $\mathcal{X}$. The notion of a diskcyclic operator was introduced by Zeana [13]. An operator T is called diskcyclic if there is a vector $x \in \mathcal{X}$ such that its disk orbit $\mathbb{D}Orb(T, x)$ is dense in $\mathcal{X}$;

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such a vector x is called diskcyclic for T. For more information about these operators, the reader may refer to [2,4,5].

In 2011, Madore and Martínez-Avendaño [9] considered the density of the orbit in a non-trivial subspace instead of the whole space, this phenomenon is called the subspace-hypercyclicity. An operator is called $\mathcal{M}$-hypercyclic or subspace-hypercyclic for a subspace $\mathcal{M}$ of $\mathcal{X}$ if there exists a vector such that the intersection of its orbit and $\mathcal{M}$ is dense in $\mathcal{M}$. They proved that subspace-hypercyclicity is an infinite dimensional phenomenon. Also, they asked whether the inverse of an invertible subspace-hypercyclic operator is again subspace-hypercyclic. This problem is still open. For more information on subspace-hypercyclicity, one may refer to [1,8,10].

In 2012, Xian-Feng et al. [12] defined the subspace-supercyclic operator as follows: An operator is called $\mathcal{M}$-supercyclic or subspace-supercyclic for a subspace $\mathcal{M}$ of $\mathcal{X}$ if there exists a vector such that the intersection of its scaled orbit and $\mathcal{M}$ is dense in $\mathcal{M}$.

Since both subspace-hypercyclicity and subspace-supercyclicity were studied, it is natural to define and study subspace-diskcyclicity. In the second section of this paper, we introduce the concept of subspace-diskcyclicity and subspace-disk transitivity. We show that not every subspace-diskcyclic operator is diskcyclic. We give the relation between different kinds of subspace-cyclicity. In particular, we give a set of sufficient conditions for an operator to be subspace-diskcyclic. We use these conditions to give an example of a subspace-diskcyclic operator which is not subspace-hypercyclic. Also, we give an example of a subspace-supercyclic operator that is not subspace-diskcyclic. Moreover, we give a simple example to show that the inverses of subspace-diskcyclic operators do not need to be subspace-diskcyclic which answers the corresponding question in [12, Question 1] for subspace-diskcyclicity. As a consequence, we show that every finite dimensional Banach space supports subspace-diskcyclic operators, which is not true for subspace-hypercyclicity.

2. Main results

In this paper, all Banach spaces $\mathcal{X}$ are infinite dimensional (unless stated otherwise) and separable over the field $\mathbb{C}$ of complex numbers. All subspaces of $\mathcal{X}$ are assumed to be non-trivial linear subspaces and topologically closed, and all relatively open sets are assumed to be non-empty. We will denote the closed unit disk by $\mathbb{D}$, the open unit disk by $\mathbb{U}$ and the set of all bounded linear operators on $\mathcal{X}$ by $\mathcal{B}(\mathcal{X})$.

Definition 2.1. Let $T \in \mathcal{B}(\mathcal{X})$, and let $\mathcal{M}$ be a subspace of $\mathcal{X}$. Then T is called a subspace-diskcyclic operator for $\mathcal{M}$ (or $\mathcal{M}$-diskcyclic, for short) if there exists a vector x such that $\mathbb{D}Orb(T,x) \cap \mathcal{M}$ is dense in $\mathcal{M}$; such a vector x is called a subspace-diskcyclic (or $\mathcal{M}$-diskcyclic, for short) vector for T.

Let $\mathbb{D}C(T,\mathcal{M})$ be the set of all $\mathcal{M}$-diskcyclic vectors for T, that is

$$\mathbb{D}C(T,\mathcal{M}) = \{x \in \mathcal{X} : \mathbb{D}Orb(T,x) \cap \mathcal{M} \text{ is dense in } \mathcal{M}\}.$$

Let $\mathbb{D}C(\mathcal{M},\mathcal{X})$ be the set of all $\mathcal{M}$-diskcyclic operators on $\mathcal{X}$, that is

$$\mathbb{D}C(\mathcal{M},\mathcal{X}) = \{T \in \mathcal{B}(\mathcal{X}) : \mathbb{D}Orb(T,x) \cap \mathcal{M} \text{ is dense in } \mathcal{M} \text{ for some } x \in \mathcal{X}\}.$$

The next example shows that subspace-diskcyclicity does not imply diskcyclicity.