Engineering Geology 220 (2017) 256–265

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Quantification of bulk form and angularity of particle with correlation of shear strength and packing density in sands

Hyoung Suk Suh [a], Kwang Yeom Kim [b], Junhwan Lee [c], Tae Sup Yun [d,*]

[a] Graduate Student, School of Civil and Environmental Engineering, Yonsei University, Seoul 03722, Republic of Korea
[b] Research Fellow, Korea Institute of Civil Engineering and Building Technology, Goyang 10223, Republic of Korea
[c] Professor, School of Civil and Environmental Engineering, Yonsei University, Seoul 03722, Republic of Korea
[d] Associate Professor, School of Civil and Environmental Engineering, Yonsei University, Seoul 03722, Republic of Korea

ARTICLE INFO

Article history:
Received 11 December 2016
Received in revised form 12 February 2017
Accepted 16 February 2017
Available online xxxx

Keywords:
Particle shape
Sphericity
Roundness
Friction angle
Void ratio

ABSTRACT

The study presents the quantification of shape parameters in sands. Natural sands, crushed sands, and glass beads are subjected to 2D microscopic and 3D X-ray computed tomographic imaging. Parameters of sphericity, elongation and slenderness are selected for analyzing the bulk forms and roundness is selected to quantify the angularity. Relationship among 2D shape parameters confirms that sphericity, elongation and slenderness are independent with roundness. Critical state friction angles are obtained by a direct shear test and void ratio ranges are measured as well. Both sphericity and roundness denote the strong linearity with void ratio range ($e_{max} - e_{min}$) bounded 0.15 and critical state friction angle (ϕ_{cs}) delineated by 20° at the unity, emphasizing that readily computable sphericity is sufficient to estimate properties of sands even without roundness. The multiple 2D projections of 3D images and their correlation for different orientation support that either bulk form or angularity in 2D images are acceptable enough to establish correlations between shape parameters and properties in sands. It implies that 2D quantification of particle shape is rational and can be used to approximate soil properties without conducting the laboratory experiments.

1. Introduction

The irregularity of particle shape is in general described by bulk form, angularity (e.g., particle-scale smoothness), and surface texture (sub-particle roughness) depending on observation scales (Barrett, 1980; Mitchell and Soga, 2005; Rodriguez et al., 2013). Quantification of particle shape can be achieved by introducing dimensionless shape parameters such as: sphericity to quantify the bulk form, and roundness and roughness to quantify the angularity and surface texture. However, due to the difficulties and complexities of measuring the surface roughness, most shape analysis focuses on bulk form and angularity (Cho et al., 2006; Zheng and Hryciw, 2015). It has been known that bulk form and angularity are independent shape parameters while both properties phenomenologically increase with decreasing irregularity although they are largely scattered and not proportional (Cho et al., 2006; Hayakawa and Oguchi, 2005). Therefore, it is desired to directly compare shape parameters at different observation scales (e.g., *sphericity* for bulk form and *roundness* for angularity). The origin, history of transportation, deposition, and production process naturally determine the particle shape in sand, which is in turn strongly correlated with index and geomechanical properties. Previous studies revealed that void ratio range (e.g., $e_{max} - e_{min}$) and compressibility increase with increasing irregularity of particles (Cavarretta et al., 2010; Cho et al., 2006; Jia and Williams, 2001; Shin and Santamarina, 2013). Round particles tend to have higher thermal conductivity than irregular particles under densification and loading because of the increase of inter-particle contact area and contact quality (Yun and Santamarina, 2008). Similarly, α-factor and β-exponent in shear velocity-stress relationship increase and decrease, respectively, with increasing particle angularity (Lee and Santamarina, 2005). Particle irregularity also affects the particle mobilization and resultant friction angles at large strain that are attributed to particle rotation, frustration and contact slippage (Cho et al., 2006; Kim et al., 2016; Shin and Santamarina, 2013; Yasin and Safiullah, 2003). These observations reside in three dimensional mechanisms while most efforts to quantify the particle shape and to correlate it with other properties of interests have been done either by semi-quantitative charts developed by Krumbein and Sloss (1963) and Powers (1953) or by two dimensional inspections including fractal approaches (Arasan et al., 2011; Cavarretta et al., 2010; Cho et al., 2006; Santamarina and Cho, 2004; Shin and Santamarina, 2013; Vallejo and Zhou, 1995; Vesga and Vallejo, 2010; Yang and Luo, 2015). Recent advances in 3D imaging technology and numerical simulation methods often allow the accurate quantification of 3D analysis for granular materials (Druckrey and Alshibli, 2016; Erdogan et al., 2006; Fonseca et al.,

---

[*] Corresponding author.
E-mail addresses: hssuh@yonsei.ac.kr (H.S. Suh), kimky@kict.re.kr (K.Y. Kim), junlee@yonsei.ac.kr (J. Lee), taesup@yonsei.ac.kr (T.S. Yun).

http://dx.doi.org/10.1016/j.enggeo.2017.02.015

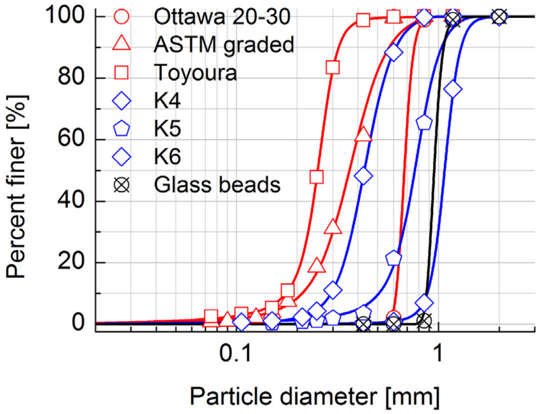

Fig. 1. Grain size distribution curves of sand specimens fitted by Fredlund et al. (2000). Reddish symbols indicate natural sands, bluish symbols denote artificially crushed sands and black symbol indicates the glass beads hereafter.

2012; Kim et al., 2016; Sun et al., 2014; Zeng et al., 2015; Zhao and Wang, 2016). Yet, the applicability of 3D shape parameters and correlation with geomechanical properties still need further investigation. We therefore investigate the validity and applicability of each shape parameters correlated with critical state friction angle and void ratio range in both 2D and 3D with the aid of microscopic and X-ray computed tomographic imagings.

2. Materials and methods

2.1. Materials

Seven sand samples are selected for analyzing shape parameters and measuring friction angles; three natural sands (Ottawa 20–30, ASTM graded, and Toyoura), three artificially crushed K-series sands (K4, K5, and K6) and spherically shaped glass beads. The measured specific gravity ranges from 2.64 to 2.68 (ASTM D854, 2014) and the range of void ratio (e.g., $e_{max} - e_{min}$) varies from 0.18 to 0.38 (ASTM D4253, 2002; ASTM D4254, 2002). The X-ray diffraction analysis (XRD) analysis indicates that specimens are mainly comprised of quarzitic mineral. The grain size distribution curves for tested specimens are shown in Fig. 1, categorized as SP according to the unified soil classification system (USCS). Table 1 summarizes the basic index properties. The similar mineralogy and uniformity warrant that the grain size and mineralogy effects can be negligible for further correlating shape parameters with other properties.

2.2. Direct shear test

The critical state friction angle denotes the frictional behavior at post-failure in large strain and is independent of initial soil condition such as void ratio. Therefore, the specimen was housed in the 60 mm diameter shear box without controlling the relative density. Housed specimen was horizontally displaced with 1 mm/min under normal forces of 50, 100 and 150 kPa until the post-peak failure was attained (ASTM D3080, 1998). It is noted that we selected only the friction angle among the shear strength parameters (e.g., friction angle and cohesion) because the tested sand was in dry condition that minimizes the cohesion. Thus, friction angles at critical state were then computed with the assumption of no cohesion, as summarized in Table 1.

Table 1
Measured properties of sand specimens used in this study.

Sand type	Index properties					Friction angle
	G_s	D_{50} [mm]	C_u	e_{max}	e_{min}	ϕ_{cs} [°]
Ottawa 20–30	2.65	0.68	1.12	0.74	0.50	27.6
ASTM graded	2.65	0.36	1.86	0.82	0.50	28.6
Toyoura	2.67	0.25	1.49	0.97	0.63	32.7
K4	2.68	1.07	1.26	1.08	0.71	39.0
K5	2.66	0.76	1.59	1.07	0.69	37.2
K6	2.64	0.43	1.56	1.04	0.66	37.4
Glass beads	2.51	0.95	1.10	0.72	0.54	21.6

2.3. Image acquisition

A total of 350 sand particles (e.g., 50 particles for each sand) is subjected to 2D images by using microscopic digital camera (MDX300, Lanoptil Tech. Ltd.) followed by binarization by flood-fill algorithm and Otsu's thresholding (Gonzalez and Woods, 2002; Otsu, 1979). The number of pixels for each average particle radius ranges from 150 to 500 depending on selected particle sizes. The 3D images of packed sands in a glass cylinder with 10 mm diameter are taken by X-ray CT imaging (PCT-G3, SEC Ltd.). The gathered 16-bit sliced images are stacked along the height and individual sand particles are segmented by the series of image processing techniques with binarization, partial erosion-dilation, and flood-fill algorithm (Al-Rousan et al., 2007; Fonseca et al., 2012; Gonzalez and Woods, 2002; Otsu, 1979). Each voxel has 20 μm and the well segmented 50 particles for each sample are randomly selected. Note that the number of sand particles in this study seems abundant for further analysis compared with previous studies (i.e., total 30 particles in Cho et al., 2006).

2.4. Form descriptors

We herein select three shape parameters of sphericity (SP), elongation (EG) and slenderness (SD) to quantify the bulk form of particle in both 2D and 3D. Fig. 2 schematically illustrates shape parameters with definitions. Note that the term of '*form descriptor*' is used for three parameters hereafter to distinguish them from '*angularity descriptor*' (e.g., roundness). Form descriptors are expressed by dimensionless number (e.g., [L/L]) and are applicable to both 2D and 3D, signifying the bulk morphology rather than surface texture. The sphericity SP indicates the degree of resemblance of an object to a circle (e.g., a sphere for 3D) and is most widely used due to its simple and various definitions (Aschenbrenner, 1956; Folk, 1955; Krumbein, 1941; Krumbein and Sloss, 1963; Wadell, 1933; Zheng and Hryciw, 2015). SP used in this study is defined as the ratio of the diameter of a circle having equivalent area (e.g., diameter of a sphere having equivalent volume for 3D) to the diameter of a circumscribing circle (e.g., diameter of a circumscribing sphere for 3D). Both elongation EG and slenderness SD share the similar concepts of how much particles are elongated but differently defined as the ratio of the longest to shortest length from the centroid to the particle surface and as the ratio of the length of the longest to short axis of fitted ellipse (e.g., fitted ellipsoid for 3D) respectively (Koo and Heng, 2001; Mora and Kwan, 2000; Wentworth, 1923).

The insufficient discretization of particle images can lead to an inaccurate estimation as highlighted in Fig. 3. Although form descriptors are supposed to be the unity for a circle in 2D and a sphere in 3D, estimated values are deviated with the number of pixels per radius less than 100 underscoring the significance of image resolution for reliable assessment. It is noted that the resolution of the image highly depends on the configured resolution of the imaging device, some relatively small particles are not able to achieve sufficient resolution. Therefore, the segmented particles in 3D X-ray images whose resolution was less than 100 voxels/radius were improved by cubic interpolation of boundary surface voxels to achieve acceptable resolution. The average values of estimated form descriptors are summarized in Table 2 with exemplary images. The spherically shaped glass bead exhibits the values close to the unity. Three crushed sands show low values while those of natural sands are high, naturally reflecting the nature of their origin. Note that shape parameters in 3D have a wider range of values than those in 2D.