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An upper bound to multiscale roughness-induced adhesion enhancement

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A R T I C L E I N F O

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A B S T R A C T

Recently Guduru and coworkers have demonstrated with neat theory and experiments that both increase of strength and of toughness are possible in the contact of a rigid sphere with concentric single scale of waviness, against a very soft material. The present note tries to answer the question of a multiscale enhancement of adhesion, considering a Weierstrass series to represent the multiscale roughness, and analytical results only are used. It is concluded that the enhancement is bounded for low fractal dimensions but it can happen, and possibly to very high values, whereas it is even unbounded for high fractal dimensions, but it is also much less likely to occur, because of separated contacts.

1. Introduction

Guduru and collaborators (Guduru [4], Guduru & Bull [5], Waters et al. [10]) have recently considered a model in which a sphere has a superposed waviness, as defined by the axisymmetric form

$$f(r)=\frac{r^2}{2R}+A\left(1-\cos\frac{2\pi r}{\lambda}\right) \tag{1}$$

i.e. with concentric waviness, where R is the sphere radius, λ is wavelength of roughness (see an example in Fig. 1). Guduru also shows that similar results are obtained if a plane roughness is assumed, similar to the function above with an x-coordinate rather than r. Guduru shows that very significant (one order of magnitude) increase of strength as well as toughness can be obtained by adding roughness, i.e. with respect to the smooth case. It should be immediately remarked that Jin et al. [6] have since then shown that some of the enhancement obtained by Guduru is specific to this assumption (either axisymmetric or purely 1D roughness), and therefore we may expect much less enhancement for, say, random roughness. However, we should also remark that Kesari et al. [8] used this otherwise rather artificial model to interpret depth-dependent hysteresis (DDH) in adhesive contacts of surfaces with apparently random roughness.

The concentric waviness permits a quite simple exact axisymmetric analysis, assuming a simply connected contact area develops. Already for a single waviness as in Guduru [4], there are some limitations for this solution to hold, as clearly for "sufficiently" large amplitude of roughness a realistic solution will show some separated contacts. Also, Waters et al. [10] have clarified that much of the enhancement comes from the assumption of JKR regime, and therefore one needs to check also the "Tabor parameter".

We shall here try to repeat some of the Guduru [4] aspects of the solution, in the context of a multiscale roughness, as it is more likely to occur in practical cases, using for simplicity a Weierstrass series instead of a single sinusoid, which was used in related contexts in Ciavarella et al. [3] without adhesion for the fully separated regime, and by Afferrante et al. [1] with adhesion, but with limited results concerning loading phase. Specifically, we assume

$$f(r)=f_0(r)+g_0\sum_{n=0}^{\infty}\gamma^{(D-2)n}\cos\left(2\pi\gamma^n r/\lambda_0\right) \tag{2}$$

where g_0 and λ_0 are length scales representing amplitude and wavelength at scale 0, whereas $f_0(r)$ is a "smooth profile" defining function, which is a convex punch for example $f_0(r)=\frac{r^2}{2R}$—we introduce this to avoid having to deal with a fully periodic surface, for which the "smooth" behaviour is itself more difficult to define. If $\gamma>1$ and $D>1$, Eq. (2) defines, in a plane section, a plane fractal surface of fractal dimension D (the real surface dimension will be one unit higher), where we have

$$g_n=g_0\gamma^{(D-2)n},\quad \lambda_n=\lambda_0\gamma^{-n} \tag{3}$$

and hence the radius at given scale n is $R_n=\frac{1}{g_n}\left(\frac{\lambda_n}{2\pi}\right)^2=\frac{1}{g_0}\frac{\lambda_0^2}{4\pi^2}\gamma^{-Dn}$.

Fig. 2 plots some examples of rough spheres so produced. Notice that the roughness may equally be present in the other body, although Guduru for his experiments considered a rough rigid sphere against an elastic nominally flat material.

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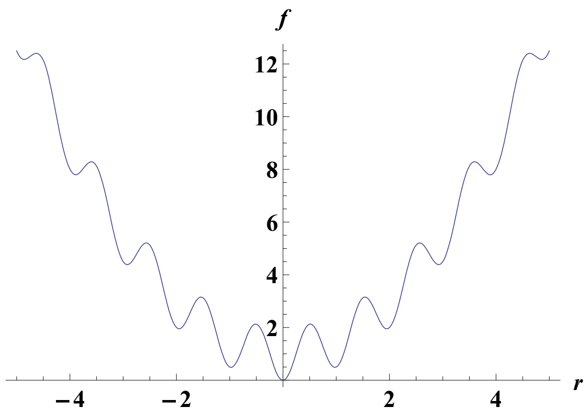

Fig. 1. The Guduru sphere in Eq. (1) for $R=\lambda=A=1$. $r<0$ used for plotting purposes.

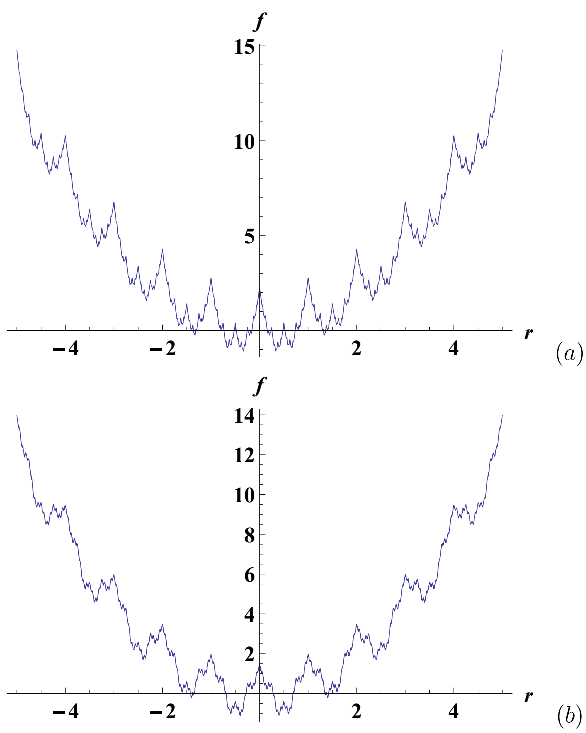

Fig. 2. The Weierstrass sphere (Eq. (2)) for $R=g_0=\lambda_0=1$, $D=1.2$. For (a) $\gamma=2$ and (b) $\gamma=4$. $r<0$ used for plotting purposes.

2. Some results

Waters et al. [10] give a good summary of Guduru's theory and experiments: it is shown that the load oscillates when it crosses a crest of a wave, and this results in highly "wavy" curves. We will not give a detailed account of this theory, as we shall instead concentrate on an asymptotic expansion solution (which permits, by joining all the minima and maxima of the resulting function, also to obtain an "envelope" solution) given by Kesari et al. [9] for a small wavelength, in particular $\lambda \ll a$, where a is the contact area radius.

Kesari et al. [9] suggest that if roughness is described by a function $\lambda_0 \varrho(r/\lambda_0)$, where the dimensionless function $\varrho(r/\lambda_0)$ can be expanded in Fourier series. Here, we shall use directly the Kesari result as a special case for the Weierstrass series in order to get deterministic results for the maxima and minima. Weierstrass is in fact a restricted form of Fourier series as we shall consider γ as integer and

$$\varrho(\xi) = \sum_{n=0}^{\infty} a_n \cos\left(2\pi\gamma^n \xi\right) \tag{4}$$

According to Kesari et al. [9] expansion, the equilibrium curves are described by load $P_K(a)$ and approach $h_K(a)$

$$P_K(a) = P_M(a) - E^* \sqrt{2\pi a^3 \lambda_0}\, \rho(a/\lambda_0) \tag{5}$$

$$h_K(a) = h_M(a) - \sqrt{\frac{\pi a \lambda}{2}}\, \rho(a/\lambda_0) \tag{6}$$

where E^* is plane strain elastic modulus and a is the contact radius, $h_M(a), P_M(a)$ correspond to the smooth profile solution, and for $\xi = a/\lambda_0$, the function $\rho(\xi)$ is given by

$$\rho(\xi) = \sum_{n=0}^{\infty} \sqrt{2\pi\gamma^n}\left[-a_n \sin\left(2\pi\gamma^n \xi - \frac{\pi}{4}\right)\right] \tag{7}$$

Guduru's case is recovered when $a_0 = A/\lambda = A/\lambda_0$, and the macroscopic shape $f_0(r)$ is Hertzian parabola. To find the envelope, one simply needs to take the maxima and minima of the equilibrium curve, which are trivial for a single sinusoid. In fact, in this case

$$P_K(a) = P_M(a) \pm 2\pi E^* \frac{A}{\lambda_0} \sqrt{a^3 \lambda_0} \tag{8}$$

$$h_K(a) = h_M(a) \pm \pi \frac{A}{\lambda_0} \sqrt{a \lambda_0} \tag{9}$$

Before proceeding further, let us notice that an interesting feature emerges in general, and that is that the smooth profile solution $h_M(a), P_M(a)$ contains a profile-independent contribution (which essentially is the flat punch solution term in the JKR model [7]), and a profile dependent part $h_{M,profile}(a), P_{M,profile}(a)$. With this separation, using 2.12a, 2.13a of Kesari et al. [9], one can derive at the quite general expressions for the Weierstrass series roughness

$$P_K(a) = P_{M,profile}(a) - a^{3/2}\sqrt{8\pi w E^*}\left(1 \pm \frac{1}{\alpha_0\sqrt{\pi}} \sum_{n=0}^{\infty} \sqrt{\gamma^n+1}\left[\gamma^{(D-2)n} \sin\left(2\pi\gamma^n \xi - \frac{\pi}{4}\right)\right]\right) \tag{10}$$

$$h_K(a) = h_{M,profile}(a) - a^{1/2}\sqrt{\frac{2\pi w}{E^*}}\left(1 \pm \frac{1}{\alpha_0\sqrt{\pi}} \sum_{n=1}^{\infty} \sqrt{\gamma^n+1}\left[\gamma^{(D-2)n} \sin\left(2\pi\gamma^n \xi - \frac{\pi}{4}\right)\right]\right) \tag{11}$$

where

$$\alpha_0 = \sqrt{\frac{2 w \lambda_0}{\pi^2 E^* g_0^2}} \tag{12}$$

is the parameter Johnson (1995) introduced for the JKR adhesion problem of a nominally flat contact with a single scale sinusoidal waviness of amplitude g_0 and wavelength λ_0.

In the general case, if we had used a Fourier representation of roughness, we would not have known how the maxima and minima of the various Fourier components could combine. But as here we are considering a Weierstrass series and we can take $\gamma \gg 1$, then the maxima and minima simply sum algebraically, leading to the envelope (assuming $\sqrt{\gamma^n+1} \simeq \gamma^{n/2}$)

$$P_{K,env}(a) = P_{M,profile}(a) - a^{3/2}\sqrt{8\pi w E^*}\left(1 \pm \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{1}{\alpha_n}\right) \tag{13}$$

$$h_{K,env}(a) = h_{M,profile}(a) - a^{1/2}\sqrt{\frac{2\pi w}{E^*}}\left(1 \pm \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{1}{\alpha_n}\right) \tag{14}$$

where we introduced a scale-dependent Johnson parameter