Tribology International 93 (2016) 446–450

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

Non-equilibrium statistical approach to friction models

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ARTICLE INFO

Article history:
Received 8 January 2015
Received in revised form
26 July 2015
Accepted 6 August 2015
Available online 21 September 2015

Keywords:
Spring-block model
Burridge–Knopoff model
Statistical fluctuation
Computational step number

ABSTRACT

A geometric approach to the friction phenomena is presented. It is based on the holographic view which has recently been popular in the theoretical physics community. We see the system in one-dimension-higher space. The heat-producing phenomena are most widely treated by using the non-equilibrium statistical physics. We take 2 models of the earthquake. The dissipative systems are here formulated from the geometric standpoint. The statistical fluctuation is taken into account by using the (generalized) Feynman's path-integral.

1. Introduction

The system we consider consists of the huge number of particles (blocks) and the size of the constituent particles is the *mesoscopic*-scale. It is lager than $50\,\text{nm} = 5\times10^{-8}$ m and is far bigger than the atomic scale ($\sim 10^{-10}$ m). It is smaller than or nearly equal to the optical microscope scale ($\sim 10^{-6}$ m) in the branches such as the soft-matter physics, the nano-science physics and the biophysics. The larger end of the mesoscopic (length) scale depends on each phenomenon. For the earthquake it is about 10^{-4} m.

The physical quantities, such as velocity, energy and entropy, are the *statistically-averaged* ones. It is not obtained by the deterministic way like the classical (Newton) mechanics. *Renormalization* phenomenon occurs not from the quantum effect but from the statistical fluctuation due to the uncertainty caused by the following facts. Firstly each particle obeys the Newton's law with different *initial conditions*. The total number of particles, N, is so large that we do not or can not observe the initial data. Usually we do not have interest in the trajectory of every particle and do not observe it. We have interest only in the macroscopic quantities: *total energy* and *total entropy* are the most important ones. Secondly, in the real system, the size and shape differ particle by particle. We regard the randomness as a part of fluctuation. Finally the models, presented in the following, contains discrete parameters (t_n in Section 2 and y_n in Section 3). As far as the discreteness is kept (in the case that we no not take the continuous limit), the quantities determined by the minimal principle include the inevitable ambiguity which is regarded as a part of fluctuation.

After the development of the string and D-brane theories [1,2], one general relation, between the 4-dimensional(4D) conformal theories and the 5D gravitational theories, was proposed. The 5D gravitational theories are asymptotically AdS_5 [3–5]. The proposal claims the quantum behavior of the 4D theories is obtainable by the classical analysis of the 5D gravitational ones. The development along the extra axis can be regarded as the renormalization flow. This approach (called AdS/CFT) has been providing non-perturbative studies in several branches: quark-gluon plasma physics, heavy-ion collisions, non-equilibrium statistical mechanics, superconductivity, superfluidity[6,7]. Especially, as the most relevant to the present work, the connection with the hydrodynamics is important [8]. When a black hole is given a perturbation, the effect decays as the relaxation phenomenon. The transport coefficients, such as viscosities, speed of sound, thermal conductivity, are important physical quantities.

We take, in Section 3, Burridge–Knopoff model for the earthquake analysis [9,10]. It was first introduced by Burridge and Knopoff [11]. Carlson, Langer and collaborators performed a pioneering study of the statistical properties [12,13]. Further development was reviewed in Ref. [14].

We exploit the *computational step number n* instead of (usual) time. The step flow is given by the discrete Morse flows theory [15,16]. In the first model (Section 2), we adopt this step-wise approach for the *time*-development. The time variable is introduced as $t_n = nh(h:\text{time}-\text{interval unit})$. In the second model (Section 3), we take the approach for the *space*-propagation. The position variable is introduced as $y_n = na(a:\text{space}-\text{interval unit})$. The non-equilibrium dissipative system is recently formulated

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http://dx.doi.org/10.1016/j.triboint.2015.08.006
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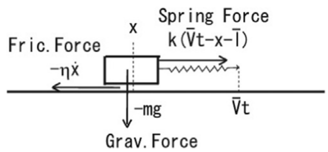

Fig. 1. The spring-block model (3).

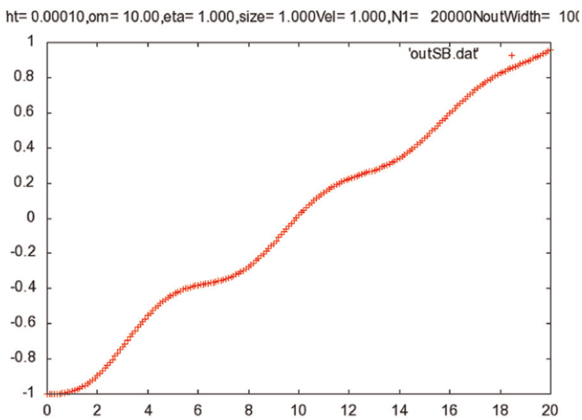

Fig. 2. Spring-block model, movement, $h=0.0001$, $\sqrt{k/m}=10.0$, $\eta/m=1.0$, $\overline{V}=1.0$, $\overline{\ell}=1.0$, total step no=20,000. The step-wise solution (2) correctly reproduces the analytic solution: $x(t)=e^{-\eta' t/2}\,\overline{V}\{(\eta'^2/2\omega^2-1)(\sin\Omega t)/\Omega+(\eta'/\omega^2)\cos\Omega t\}-\overline{\ell}+\overline{V}(t-\eta'/\omega^2), \Omega=(1/2)\sqrt{4\omega^2-\eta'^2}=9.99,\ 0\le t\le 2, x(0)=-\overline{\ell},\ \dot{x}(0)=0.$

using the discrete Morse flows theory combined with the (generalized) path-integral [17,18].

2. Spring-block model

We treat the movement of a block which is pulled by the spring which moves at the constant speed $\overline{V}$. The block moves on the surface with friction. This is called the spring-block (SB) model. We adopt the *discrete Morse flows* method to treat this non-equilibrium system [15,16]. We take the following n-th energy function to define the step(n) flow.

$$K_n(x)=V(x)-hnk\overline{V}x+\frac{\eta}{2h}(x-x_{n-1})^2+\frac{m}{2h^2}(x-2x_{n-1}+x_{n-2})^2+K_n^0,$$
$$V(x)=\frac{kx^2}{2}+k\overline{\ell}x, \tag{1}$$

where η is the friction coefficient and m is the block mass. h is the 'time' interval parameter. x is the position of the block. The potential $V(x)$ has two terms: one is the harmonic oscillator with the spring constant k, and the other is the linear term of x with a new parameter $\overline{\ell}$ (the natural length of the spring). $\overline{V}$ is the velocity (constant) with which the front-end of the spring moves. ${K_n}^0$ is a constant which does not depend on x. The n-th step x_n is determined by the energy minimum principle: $\delta K_n(x)|_{x=x_n}=0$ with the pre-known position at the (n-1)-th, x_{n-1}, and that at the (n-2)-th, x_{n-2}.

$$\frac{k}{m}(x_n+\bar{l}-nh\overline{V})+\frac{1}{h^2}(x_n-2x_{n-1}+x_{n-2})+\frac{\eta}{m}\frac{1}{h}(x_n-x_{n-1})=0,$$
$$\omega\equiv\sqrt{\frac{k}{m}},\eta'\equiv\frac{\eta}{m}, \tag{2}$$

where $n=2,3,4,\ldots$. For the continuous *time* limit: $h\to 0, nh=t_n\to t, v_n\equiv(x_n-x_{n-1})/h\to\dot{x}, (x_n-2x_{n-1}+x_{n-2})/h^2\to\ddot{x}$, the above recursion relation reduces to the following differential equation.

$$m\ddot{x}=k(\overline{V}t-x-\overline{\ell})-\eta\dot{x}. \tag{3}$$

This is the ordinary one for the spring-block model. See Fig. 1.

The graph of movement (x_n, Eq. (2)) is shown in Fig. 2. From the graph, we see this system starts with the *stick-slip* motion and reaches the *steady state* as $n\to\infty$. Fig. 3 shows the energy change as the step flows. It shows the energy oscillates periodically and the amplitude goes down as the step goes. The physical dimensions of the parameters in Eq. (3) are listed as

$$[m]=\mathrm{M},[k]=\mathrm{MT}^{-2},[\overline{\ell}]=\mathrm{L},[\eta]=\mathrm{MT}^{-1},[\overline{V}]=\mathrm{LT}^{-1}, \tag{4}$$

where we assume that $[x]=$ L, $[t]=$ T and $[h]=$ T. (M: mass, T: time, L: length.)

Now we consider N *copies* of the one body system (2). N is sufficiently large, for example, 10^{23}(1 mol). We are *modeling* the present statistical system as follows. The N particles are "moderately" interacting each other in such way that each particle almost independently moves except that energy is exchanged. The interaction is not so strong as to break the dynamics (2). We use Feynman's path-integral method in order to take the statistical average of this N-copies system. The statistical ensemble measure will be given explicitly.

From the energy expression (1), we can read the *metric* (*geometry*) of this mechanical system.

$$\Delta s_n^2\equiv 2h^2(K_n(x_n)-K_n^0)=2\,dt^2V_1(X_n,t_n)+(\Delta X_n)^2+(\Delta P_n)^2,$$
$$V_1(X_n,t_n)\equiv V\left(\frac{X_n}{\sqrt{\eta h}}\right)-nk\sqrt{\frac{h}{\eta}}\overline{V}X_n, dt\equiv h, t_n=nh, \tag{5}$$

where $X_n\equiv\sqrt{\eta h}x_n, P_n/\sqrt{m}\equiv hv_n=(x_n-x_{n-1})$. Using this metric, we can introduce the associated *statistical ensemble* of the spring-block model [19–21].

[Statistical Ensemble 1a]

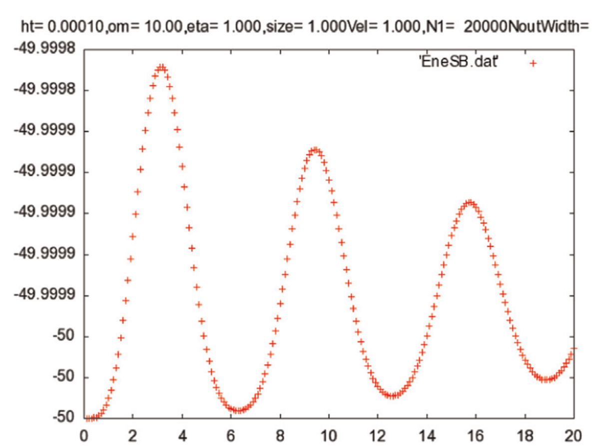

Fig. 3. Spring-block model, energy change, $h=0.0001$, $\sqrt{k/m}=10.0$, $\eta/m=1.0$, $\overline{V}=1.0$, $\overline{\ell}=1.0$, total step no=20,000.

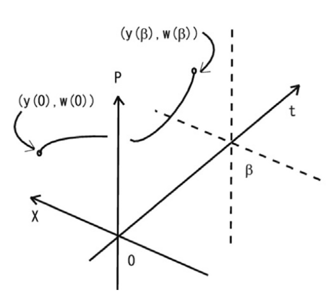

Fig. 4. The path $\{(y(t),w(t),t)|0\le t\le\}$ of line in 3D bulk space (X,P,t).