Tribology International 83 (2015) 21–32

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

Numerical analysis of leakage of elastomeric seals for reciprocating circular motion

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ARTICLE INFO

Article history:
Received 6 June 2014
Received in revised form
22 October 2014
Accepted 31 October 2014
Available online 11 November 2014

Keywords:
Cambered elastomeric seal
Reynolds equation
Reciprocating rotation
Leakage

ABSTRACT

A cambered elastomeric seal used in a novel universal joint for reciprocating circular motion is investigated. Then based on the linear elastic Hooke equations and the Reynolds equation, mathematical models of the contact pressure, film thickness and leakage with the method of variable substitution are established. The numerical analysis and simulation are performed based on the modified inverse hydrodynamic (IH) method and MATLAB numerical method. Besides, effects of various parameters on the sealing performance are investigated systematically. The simulation results verify the effectiveness and feasibility of proposed mathematical model and numerical algorithm. The results also lay the theoretical basis for the structure design and performance analysis of the seal assembly.

1. Introduction

Elastomeric seals play an important role in the field of modern industry, which are the key to guarantee the reliable performance of mechanical systems. They are widely applied in various environments of hydraulic systems, especially for static and dynamic seals with rubber sealing materials. However, leakage is a serious potential problem in hydraulic systems. As one of the most important indexes to evaluate the sealing performance, leakage depends on many factors, such as the seal material, operating temperature, driving pressure, seal compression and seal geometry. Therefore, effects of these factors on the behavior of the seal should be thoroughly understood and the leakage rate is reasonably determined, which is the primary mission of the seal design.

Leakage is closely related to the contact pressure and the film thickness at the sealing contact. Researches concerned with modeling of the contact pressure and film thickness for common sealing forms could be found in many literatures, such as O-rings [1–3], rectangular seals [4–7] and U-cups [8,9], et al. For modeling of the contact pressure at a sealing contact, it was generally computed by either the elementary stress–strain relations or assuming the plane-strains conditions [10–14]. Nikas established the sealing contact pressure of rectangular seals respectively based on the linear elastic theory [11] and the Mooney–Rivlin theory [12]. The finite element method was also applied for the sealing form with special cross-section or the purpose of high computational accuracy [15–19]. While for calculating of the film thickness, it was mainly on the basis of the elastohydrodynamic theory [20], which was represented by one and two dimensional forms of the Reynolds equation in actual application. As common seals are usually axisymmetric, it means that the leakage mainly occurs along the symmetry axis. Therefore, one-dimensional Reynolds equation is widely applied. When the fluid transportation transversely to the direction of motion is taken into consideration, the form of two-dimensional Reynolds equation is used. Nonetheless, the computation is much more complex [10]. Despite the diversity of related subject, exiting literatures on the elastomeric seals mainly focus on the static seals and reciprocating motion for the O-rings or rectangular rings. Elastomeric sealing studies for those with special cross section are rarely seen for their irregular appearance and proprietary nature, such as the cambered elastomeric seal. The significant contributions in this aspect can be found in the literatures of Yang [21] and Johannesson [22].

Due to the coupling between the contact pressure and film thickness in the Reynolds equation, one method of solving the Reynolds equation was based on the known (or assumed) sealing contact pressure to calculate the film thickness. Müller [23] calculated the film thickness and leakage with the experimental measured contact pressure. White and Denny [24] assumed that the contact pressure distribution was parabolic to solve the film thickness. Generally, accuracy became the critical issue for this method. With the development of numerical theory, iterative methods were used, such as the Runge–Kutta method [25] and the Petrov–Galerkin method [26]. The core idea of iterative methods was to establish the contact pressure by the finite

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http://dx.doi.org/10.1016/j.triboint.2014.10.022
0301-679X/

Nomenclature

a, b, l_z, R_2 dimensions of elastomeric seal before installation (Fig. 2), mm
a', b' dimensions of elastomeric seal after installation (Fig. 2) (mm)
A, B pressure–density coefficients (Pa^{-1})
d wire diameter of spring (mm)
D outer diameter of coil (mm)
E elastic modulus of the elastomeric seal (MPa)
h, h_m film thickness, $h_m=h(\theta=\theta_m)$ (Eq. (20)) (μm)
h_{min} minimum film thickness (μm)
h_0 overall film thickness (μm)
H shear modulus of spring (GPa)
m mass leakage rate of per width of elastomeric seal (Eq. (21)) (mg/(h•°))
n cycle index
N_c active coils of spring
p contact pressure (Eq. (2)) (MPa)
q_1, q_2, q, q_1', q_2', q', Q fluid leakages: q_1, q_2 (Eq. (22)), q_1',q_2' (Eq. (23)), Q (Eq. (24)), $q=q_1-q_2$; $q'=q_2'-q_1'$; $Q=q+q'$ (mg/h)
r_0, d_0, R_1 dimensions of universal joint pin (Fig. 2) (mm)
R-square coefficient of determination
s, t, T intermediate variables, (Eq. (13))
SSE sum of squares due to error
T_m, T_n $T_m=T(\theta_m)$ (Eq. (15)), $T_n=T(\theta_n)$ (Eq. (18))
w contact angular velocity of the universal joint pin relative to elastomeric seal (rad/s)
Z experimental constant (Eq. (7)), dimensionless
α, β anticlockwise (or clockwise) rotation angle of the universal joint pin relative to the elastomeric seal (Fig. 3) (°)
γ, φ central angles (Fig. A1) (°)
δ_x, δ_y, δ_z interferences of elastomeric seal in Cartesian coordinate system (Eq. (3)) (mm)
δ_z' longitudinal interference of elastomeric seal caused by spring deformation (mm)
δ_θ z-interference of the elastomeric seal in polar coordinate system (Eq. (5)) (mm)
Δz spring deformation (Eq. (4)) (mm)
ε_x, ε_y, ε_z strains of the elastomeric seal in Cartesian coordinate system (Eq. (3))
ζ pressure–viscosity coefficients (Pa^{-1})
η, η_0 fluid dynamic viscosity (Eq. (6)), $\eta_0=\eta(p=0)$ (Pa s)
θ_m polar angle coordinate of the extreme point for T in polar coordinate system ($dT(\theta_m)/d\theta=0$) (°)
θ_n polar angle coordinate of the inflection point for T in polar coordinate system, $d^2T(\theta_n)/d\theta^2=0$, computed by Eq. (19) (°)
λ, G Lamé constants (MPa)
Λ lambda ratio: $\Lambda=h_0/\sigma$
μ Poisson's ratio of the elastomeric seal
ξ film thickness ratio: $\xi=h_{min}/\sigma$
ρ, ρ_0 fluid mass density (Eq. (8)), $\rho_0=\rho(p=0)$ (kg/m^3)
σ_1, σ_2, σ root mean square roughnesses, $\sigma=(\sigma_1^2+\sigma_2^2)^{1/2}$ (μm)

subscripts

i coordinate sequence
j basic cycle

element theory and further to solve the film thickness iteratively until the solution was converged. And it was adopted by Field and Nau [4] to calculate the contact pressure, film thickness and the leakage of rectangular seals. The significant advantage of the iteration methods was the high precision of final solution. However, serious problems including numerical stability and inconsistency existed due to the high nonlinearity of the Reynolds equation. In order to tackle the numerical stability problem caused by coupling of the contact pressure and film thickness, a numerical algorithm based on the principle of Newton iteration was developed by Ruskell [15]. Specifically, an integrodifferential equation was obtained through the combination of the elasticity equation and the Reynolds equation, which could be further solved iteratively. Nevertheless, because the contact pressure at sealing contact was solved through the cumbersome finite element analysis, efficiency was still a remained problem in solving process [10]. With improvement of the inverse hydrodynamic (IH) theory, the IH method and modified versions were gradually developed to solve the Reynolds equation [6,27–30]. However, the numerical instability of imaginary roots could quickly destroy the convergence of the final correct solution for application of the IH method. This problem was tackled effectively by Nikas [6], who proposed a modified IH method that avoided solving the cubic algebraic equation of the film thickness and could be solved by a robust numerical algorithm. The application of this method greatly improved the numerical stability and solving speed.

A cambered elastomeric seal used in a novel universal joint for reciprocating circular motion is investigated systematically in this paper. Based on the linear elastic Hooke equations and Reynolds equation, the mathematical models of the contact pressure, film thickness and leakage with the variable substitution are established. The numerical analysis and simulation are performed on the basis of the modified IH method and the MATLAB numerical tool. The effects of various parameters on the sealing performance are investigated systematically. The rest of this paper is organized as follows. The structure of the universal joint and the sealing principle of cambered elastomeric seal are briefly introduced, and then the mathematical models of contact pressure, film thickness and leakage are presented in Section 2. Section 3 describes the proposed numerical algorithm, simulation procedure, simulation results and related discussion. Finally, some important conclusions and the future work are drawn in Section 4.

2. Mathematical model

2.1. Structure of cambered elastomeric seal

In the drilling process, control for directional well trajectories is achieved via the spindle offset for the point-the-bit drilling tools. Therefore, the spindle is subject to the alternating load in practical application, which even leads to fatigue failure. According to a kind of hollow universal joint that can not only transfer torque but also adjust the drilling direction [31], as shown in Fig. 1, the problem mentioned above can be solved effectively. However, the sealing problem is one of the most critical issues. It is the cambered elastomeric seal that plays an effective role in the seal for reciprocating circular motion of the universal joint pin. The partial enlarged drawing of seal assembly is shown in the lower right corner of Fig. 1.

The section view of seal assembly is shown in Fig. 2. The cambered elastomeric seal is fixed with the external member and