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# A method for determining the optimal back-washing frequency and duration for dead-end microfiltration

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ABSTRACT

Microfiltration is used in a variety of industrial and municipal water purification settings where one of the main concerns is fouling from the particulate matter that is removed from the water. Our focus has been on developing a unified model that captures fouling behavior in a consolidated manner rather than describing individual blocking regimes using power law models. The unified model provides greater insights into fouling mechanisms so that a deeper understanding of flux decline can be obtained. Moreover, by characterizing both forwards and backwashing behavior together, mathematical theory is available to develop strategies that increase the effectiveness of microfiltration in conjunction with backwashing used to regenerate the filter. We present a very simplified model that was developed to provide details regarding the mathematical analysis and how optimal control theory can be used to predict the timing and duration of backwashing that will optimize the overall water flow through the membrane. We use optimal control theory to derive an analytic solution to the optimal problem and develop a strategy to implement the solution. The model estimates of forward operation are compared with experimental data for constant pressure filtration and indicate that the model is able to capture the basic processes. More interestingly, the optimal control solution and proposed implementation strategy are consistent with empirical demonstrations but provide mathematical evidence that the flux may be increased dramatically by precise timing of the forward and backwashing cycles. Model predictions can be evaluated during pilot-testing that often precedes microfilter regulatory approval and plant design.



## 1. Introduction

Microfiltration (MF) and ultrafiltration (UF) membranes are commonly installed to purify municipal water and wastewater because of their excellent capabilities to remove difficult to disinfect protozoa, bacteria, and turbidity [1]. MF/UF are also used as pretreatment processes to reduce fouling of reverse osmosis and nanofiltration membranes during desalination, surface water treatment, and water reuse [2–4]. In spite of the widespread implementation of MF/UF, fouling continues to be an important limitation for their continued growth. Various strategies including source water conditioning (i.e. pretreatment), periodic chemically enhanced backwashing, chemical cleaning, and routine hydraulic backwashing are employed to combat MF/UF fouling [5–9].

Early quantitative models of backwashable systems and their experimental verification largely focused on crossflow filtration of biological feed waters and/or frequent flow reversals (order of seconds) [10–13]. In contrast, municipal MF/UF systems are operated in the dead-end configuration, filter a heterogeneous feed water that includes microorganisms, suspended materials, natural organic matter, and dissolved ions, and are backwashed approximately every 10–30 min [14,15]. Under these conditions, hydraulic backwashing does not completely remove the deposited foulants, forcing the effective permeability of the membrane to decline over an extended time frame (order of weeks or months) [16,17].

Most states in the United States mandate an on-site pilot-scale study to provide operational and water quality data in support of the governmental permitting process since membranes are classified as alternative filtration technologies by the Environmental Protection Agency [18]. The high cost of such investigations precludes extensive testing allowing in most cases only a highly truncated experimental matrix to provide operational data in support of system design and regulatory approval. Importantly, in most cases this field evaluation presents the only opportunity to quantify design variables such as process configuration, membrane type, operating flux, backwashing frequency, need for and type of pretreatment, and chemical cleaning interval [19]. An accurate model that captures backwashing effects on fouling kinetics will

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assist in better designing on-site experiments so that the limited time available during regulatory permitting can be better utilized to provide design data for the more optimal design of MF/UF facilities.

The principal objective of this research is to develop such a mathematical model wherein recent efforts to rigorously predict the optimal course of hydraulic backwashing have been extended to maximize the flux of water through the filter. This model is based on previous studies that aimed to describe the fouling process using dynamical systems where the flux decline was functionally related to the filtration process, accumulation of foulants, and contaminant profile (e.g. bacterial type etc.) [20,21].

## 2. Model development: incorporating forward filtration and backwashing

In this section, a simplified model of dead-end filtration is developed that includes the effects of backwashing on the filtration process. The spirit of previous investigations [20,21] is followed; however, several additions have been made to the model and several important restrictions are indicated. Most importantly, the effects of backwashing have been included in the current formulation of the model. This allows us to study methods for optimizing the fluid filtration using a combination of forward and backwards operation.

Including this effect in the model is quite novel and further supports efforts to move from empirical, power-law like fitting models to more descriptive and flexible differential equation models. In fact, under suitable assumptions (described below), it is possible to determine the optimal backwashing timing and duration analytically. This is a major step in proposing efficient techniques to improve the efficiency of dead-end filtration.

In previous models, the free-filter area depends dynamically on the bacterial foulants, exo-polymeric substances produced by the bacteria. Here the basis is a much simpler model that only includes non-biological foulants and neglects irreversible attachment. In principle, there is no requirement for these restrictions; however, the mathematical techniques that are introduced become much more complicated so the initial focus is on the simplest case. Even with these restrictions, the model captures the experimental observations quite well and is also able to make empirically novel predictions for the backwashing timing.

The model consists of a single ordinary differential equation that describes the accumulation of the foulant, $B$, on the filter. The focus is on constant pressure filtration where the pressure gradient, $\Delta P$, is constant during the operation. The pressure gradient applied during forward filtration is assumed to be the same as the gradient applied during backwashing, a method that is commonly used by many manufacturers. Thus the pressure drop is equal in magnitude, but in the opposite direction during forwards and backwards operation. The parameter $u$ controls the direction of the pressure drop. The parameter, $u$, is piecewise constant taking the value 1 during forward filtration and $-1$ during backwashing. $u(t)$ is referred to as the *control* and $u(t)\Delta P$ describes the filtration protocol and is piecewise constant throughout the filtering process, i.e. forward filtration and backwashing.

The flux of water through the filter as a function of time, $J(t)$, depends on the resistance of the filter which, in turn, depends on the constant membrane filter resistance, $R_m$ and the resistance due to the accumulating foulant, $R_b$:

$$J(t) = \frac{\Delta P}{\mu(R_m + R_b)}, \tag{1}$$

where $\mu$ is the absolute viscosity of the solution.

This is similar to the resistance in series models that have been described previously [25]. The resistance due to foulant accumulation should be a monotonically increasing function of the foulant on the filter during forward filtration. The particular details may be complicated if complete or incomplete fouling are included, so this study begins with the simplest description of the resistance due to the foulant,

$$R_b = \nu B, \tag{2}$$

where $\nu$ is the specific resistance per unit of foulant and $B$ is the foulant density on the filter. We construct a model that can incorporate both backwashing and forward filtration based on the value of the control parameter, $u$. We expect that when the filter is operated in forward filtration, foulants accumulate on the filter which subsequently affects the flux through the filter (as indicated in Eq. (1)). We take a simple model of foulant accumulation that assumes the time rate of change of the foulant density is proportional to the flux through the membrane. The constant of proportionality is denoted, $K$ and describes the amount of foulant in the bulk fluid. Therefore, when the filter is operated in forward mode, $u=1$ and

$$\frac{dB}{dt} = KJ. \tag{3}$$

During backwashing, we assume that the foulant density decreases at a rate proportional to the product of the flux and the current foulant density on the filter, $B$. The constant of proportionality, denoted $\hat{K}$, represents the effectiveness of the removal. If the foulant was irreversibly attached, $\hat{K}=0$. During backwashing, when $u=-1$, the change of foulant on the filter is described by the differential equation:

$$\frac{dB}{dt} = -\hat{K}JB, \tag{4}$$

We can combine these two modes into a single, piecewise defined differential equation representing the combined effects of accumulation and removal, when $u(t)$ is a piecewise constant function taking values of 1 and $-1$ during forward operation and backwashing, respectively. Our combined model is

$$\frac{dB}{dt} = \underbrace{\frac{(1+u(t))}{2}KJ}_{\text{fouling}} - \underbrace{\frac{(1-u(t))}{2}\hat{K}JB}_{\text{removal/backflow}}, \tag{5}$$

where $K$ and $\hat{K}$ scale the accumulation (removal) rate during forward filtration (backwashing), respectively and depend on the foulant concentration. $u$ is dimensionless, so $K$ has units of $[B]/[T][J] = \text{g m}^2\,\text{L}^{-2}$ and $\hat{K}$ has units of $1/[T][J] = \text{m}^2\,\text{L}^{-1}$. We assume that the filter is initially clean, which implies that $B(0)=0$.

Observations of backwashing indicate that, while backwashing fails to remove all accumulated foulants, it takes a far shorter time to remove the majority of the foulants from the filter than it does for the foulants to accumulate (i.e. backwashing removal occurs at a faster rate than fouling). This is incorporated into the model by assuming that $\hat{K} \gg K$.

During forward operation, $u=1$, and $dB/dt \geq 0$ and $B$ accumulates on the filter. During backwashing, $u=-1$, and $dB/dt \leq 0$ and, since $J>0$, the foulant is decreasing and tends toward zero. Because $u$ is a piecewise constant, it is either 1 or $-1$ to describe forwards or backwashing modes of operation. There are other aspects that are neglected in this model including a range of important processes such as physicochemical and biological details of the foulants, irreversible attachment, and flow effects. These are relatively straightforward to include by incorporating extracellular polymeric substances (EPS) and assuming the backwashing efficiency to depend on the EPS density (through $\hat{K}$, for example). However, as shown in Fig. 2, the forward filtration component is able to capture experimental data quite well for the