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Examining the correlations between drop size distribution parameters using data from two side-by-side 2D-video disdrometers

M. Thurai [a,*], C.R. Williams [b], V.N. Bringi [a]

[a] Colorado State University, Fort Collins, CO, USA
[b] CIRES, University of Colorado, Boulder, USA

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ABSTRACT

As part of a long-term observation campaign, over 5000 pair samples of temporally matched 1-minute averaged drop size distribution (DSD) measurements have been recorded by two side-by-side (frequently calibrated) 2D-video disdrometers. The measurement campaign was conducted in Huntsville, Alabama, over a ten-month period, and includes a variety of rain types and regimes. The datasets have been used to examine, (i) the relationship between the mass-weighted mean diameter (D_m) and the standard deviation of the mass spectrum (σ_M) without any assumption on the DSD form, and (ii) the relationship between the shape parameter (μ) and slope parameter (Λ) of the gamma form of the DSD. A number of methods were used to estimate μ and Λ, including: a normalizing procedure, the method of moments, the maximum likelihood method and the L-moment method. The physical validity of the σ_M–D_m relationship is examined by, (i) relating the estimated D_m versus the estimated σ_M from the same disdrometer datasets and, (ii) by 'cross-relating' D_m from one disdrometer with the corresponding σ_M from the second disdrometer dataset. The same procedures were repeated to examine the physical validity of the μ–Λ relation. It is shown that the transformed variable $\sigma_M' = \sigma_M D_m^{-1.65}$, which is uncorrelated with D_m, has a narrow histogram and that $\sigma_M' \approx$ constant can form a constraint which may well be applicable to other rain climatologies. For the μ–Λ relationship, the variation between μ from one unit and Λ by the second unit showed, as expected, larger scatter than using estimates from the same unit but not excessively so given that the μ estimates from the two units themselves show some instrument-to-instrument variability. While we cannot ascertain that the removal of any statistical correlations necessarily implies that the μ–Λ relation is physical, we have gone on to show that instrument limitations of accurately measuring the number concentration at the small drop end ($D < 0.5$ mm) are likely to be more important in giving rise to spuriously large μ values and related higher μ–Λ correlation, especially in light rain, and perhaps to a lesser extent in heavier rain.

1. Introduction

The drop size distribution (DSD) and its spatio-temporal variation in different rainfall types/intensities are well-known to be important in microphysical studies (e.g., Ulbrich and Atlas, 2007) as well as in developing radar-based algorithms for rain rate (R) estimation either using polarimetric, dual-frequency or profiling radars (e.g., Bringi and Chandrasekar, 2001; Kozu and Nakamura, 1991; Williams et al., 2007). It was shown by Ulbrich (1983) that the un-normalized gamma form for the DSD involving the three parameters [N_0, Λ, μ] defined in Eq. (1) can capture the natural variability of the DSD under widely varying rainfall types and intensities. The concept of normalizing the DSD was introduced by Sekhon and Srivastava (1971) in order to

* Corresponding author at: Dept. of Electrical and Computer Engineering, Colorado State University, Fort Collins, CO 80523-1373, USA.
E-mail address: merhala@engr.colostate.edu (M. Thurai).

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compare the shapes of DSDs with widely varying rainwater contents (W) which was later extended by Illingworth and Blackman (2002) and Testud et al. (2001). The latter defined N_0^* (or N_w) as a normalized intercept parameter being proportional to W/D_m^4 such that $N(D) / N_w = F(X)$ where $X = D / D_m$ and F(X) is the normalized distribution which can take the gamma form (given later in Eq. (3)) if desired. In this case the DSD parameter set is $[N_w, D_m, \mu]$ where D_m is the mass-weighted mean diameter (defined in Eq. (4)). Another approach was taken by Haddad et al. (1997) who mapped the un-normalized gamma parameters $[N_0, \Lambda, \mu]$ to an uncorrelated set of parameters $[D_m', \sigma_M', R]$ where $D_m' = D_m R^{-0.155}$ and $\sigma_M' = \sigma_M D_m^{-1.65}$ based on airborne DSD measurements. Note that σ_M is the standard deviation of the mass spectrum with respect to D_m as defined in Ulbrich and Atlas (1998) (defined in Eq. (5)).

In order to make the radar retrieval algorithms tractable, the various forms of the three parameter set are usually reduced to two parameters. Examples include setting the shape parameter $\mu = 3$ (Kozu et al., 2009), or constraining the variation of μ with Λ (Zhang et al., 2001; Vivekanandan et al., 2004) or setting $\sigma_M' \approx$ constant or only allowing it to vary within a narrow a priori selected range (Haddad et al., 1997). Such constraints are often based on disdrometer measurements of the DSD (either at surface or using airborne probes) with the three parameters being estimated using method of moments (Tokay and Short, 1996; Ulbrich and Atlas, 1998), maximum-likelihood (Wong and Chidambaram, 1985; Haddad et al., 1996) or based on the normalized-scaling approach (Testud et al., 2001; Bringi et al., 2003).

In the literature, correlations between the DSD parameters have been explored using ground-based disdrometer measurements, e.g., N_0–Λ (Tokay and Short, 1996) or μ–Λ (Zhang et al., 2003). Typically, DSD measurements obtained over 1 or 3 min have been used as input to the estimation procedures. However, some questions and ambiguities can arise from such an approach because the same DSDs from the same instrument are utilized and hence the correlations might be internally generated and artificially introduced by the estimation procedures themselves. Simulations of gamma DSDs have been used to demonstrate that correlations between DSD parameter estimates are more due to statistical correlations between the moments and not due to physical DSD variations (e.g., Chandrasekar and Bringi, 1987; Smith et al., 2009; Moisseev and Chandrasekar, 2007).

To address the question of whether or not the correlations are artificially introduced by the estimation procedures, we present here an analysis of data from two collocated (side-by-side) 2D-video disdrometers (Schönhuber et al., 2008; Kruger and Krajewski, 2002). The use of two (nearly) identical and collocated units imply that they are sampling the same underlying DSDs so that the statistical correlation between different moments that mask physical variations when DSDs from a single unit are used, can be eliminated by 'cross relating' parameter estimates between the two units. In this paper, we specifically address the questions: (i) is there a relation between D_m–σ_M, and (ii) is there a μ–Λ relation, and if so, are they physically-based or just a manifestation of the procedures used to estimate these parameters?

DSD formulations considered in this study are given in Section 2, followed by a description of datasets in Section 3. Results of the correlation analysis are presented in Section 4, together with pertinent discussions, followed by a summary in Section 5.

2. DSD characterization

The un-normalized gamma form of the DSD is given as (Ulbrich, 1983):

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D) \tag{1}$$

where N_0, Λ and μ are termed as the intercept, slope and shape parameters, respectively.

The slope parameter can be expressed in terms of the median volume diameter (D_0) and μ as (Ulbrich, 1983):

$$\Lambda = \frac{3.67 + \mu}{D_0}. \tag{2}$$

The normalized gamma form of Testud et al. (2001) is expressed as:

$$\frac{N(D)}{N_W} = f(\mu)\left(\frac{D}{D_m}\right)^{\mu} \exp\left\{-(4+\mu)\frac{D}{D_m}\right\} \tag{3}$$

where N_w and $f(\mu)$ are given in Testud et al. (2001) and not repeated herein. D_m is the mass-weighted mean diameter which is the ratio of 4th to 3rd moments of the DSD:

$$D_m = \frac{\int D^4 N(D) dD}{\int D^3 N(D) dD}. \tag{4}$$

In particular, for the gamma form, $\Lambda D_m = 4 + \mu$ (Ulbrich, 1983). The standard deviation (σ_M) of the mass spectrum with respect to D_m is defined as (Ulbrich, 1983):

$$\frac{\sigma_M}{D_m} = \sqrt{\left\{\frac{\int (D-D_m)^2 D^3 N(D) dD}{D_m^2 \int D^3 N(D) dD}\right\}}. \tag{5}$$

For the gamma form, $\sigma_M / D_m = (4 + \mu)^{-1/2}$. Note that D_m, D_0, σ_M and N_w can be estimated from measured DSDs directly by definition. Such is not the case for N_0, μ and Λ which must be estimated, for example, using the method of moments or maximum-likelihood.

3. Datasets and instrument-to-instrument variability

Data from two, frequently calibrated, side-by-side, 2D video disdrometers (abbreviated as 2DVD) in Huntsville, Alabama have been used. The instruments are referred to by their serial numbers SN16 and SN25 hereafter. SN16 is a second-generation, low-profile, unit with its sensor area close to the ground (≈30 cm). SN25 is a third generation, compact unit, also having its sensor area close to the ground, but perhaps with slightly different sensitivity to the tiny drops.

Fig. 1 shows the two instruments side-by-side, taken at the time of the installation of SN25. The procedures for optical alignment were followed and plane distance calibration was performed frequently. Additionally, sphere calibration tests were done on site for both instruments approximately every