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Forecasting daily river flows using nonlinear time series models

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SUMMARY

In the hydrology studies, it is well known that the river flows are affected by various factors, and therefore the dynamics in their associated time series are complicated and have nonlinear behaviors. In an empirical study, we investigate the capability of five classes of nonlinear time series models, namely Threshold Autoregressive (TAR), Smooth Transition Autoregressive (STAR), Exponential Autoregressive (EXPAR), Bilinear Model (BL) and Markov Switching Autoregressive (MSAR) to capture the dynamics in the Colorado river discharge time series. Least Squares (LS) and Maximum Likelihood (ML) methods are employed to estimate parameters of the models. For model comparison three criteria, namely loglikelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC) are calculated. The results show that a self-exciting TAR (SETAR) model performs better than other four competing models. To forecast future river discharge values an iterative method is applied and forecasting confidence intervals are constructed. The out-of-sample 1-day to 5-day ahead forecasting performances of the models based on ten forecast accuracy measures are evaluated. Comparing verification metrics of all models, SETAR model presents the best forecasting performance.

1. Introduction

The inherent nonlinear nature of hydrologic systems and the associated processes has been known for long (e.g., Izzard, 1966; Amorocho, 1967; Amorocho and Brandstetter, 1971; Coulibaly and Baldwin, 2005). River flow specially in snowmelt driven rivers can be characterized by several general features. As a result of the periodicity in precipitation, river flow has also strong seasonal periodicity. The seasonal cycle of river flow is asymmetric; i.e., river flow increases rapidly (usually during late winter and spring) and decreases gradually (towards the end of autumn). The fluctuations in river flow are large for large river flow and small for small river flow. It is important to note that unlike other climate components, river flow with a few exceptions may have been impacted by human activity, like damming, use of river water for agriculture, etc., a fact which makes the river flow data more difficult to study. The fluctuations in river flow are of special interest since they are directly linked to floods and droughts. There are several interesting characteristics of river flow fluctuations: (i) the river flow fluctuations have power law tails in the probability distribution (e.g., Murdock and Gulliver, 1993; Kroll and Vogel, 2002), (ii) the river flow fluctuations are long-term correlated (e.g., Hurst, 1951; Pelletier and Turcotte, 1997; Koscielny-Bunde et al., 2006), and (iii) river flow fluctuations are multifractal (e.g., Tessier et al., 1996; Pandey et al., 1998; Kantelhardt et al., 2003). These scaling laws may improve the statistical prediction of extreme changes in river flow (e.g., Bunde et al., 2004). Streamflow forecasting is of vital importance to flood mitigation and water resources management and planning. The short-term forecasting such as hourly or daily forecasting is crucial for flood warning and defense. There are a variety of available methods for forecasting streamflows, which may fall into two general classes: process-driven methods and data-driven methods (Wang, 2006). Data-driven methods mathematically identify the connection between the inputs and the outputs. One of the popular Data-driven methods is the time series models approach, which is the focus of this research. This approach is used for building mathematical models to generate synthetic, hydrological records and to forecast hydrological events (Salas, 1993). A variety of linear AR models are used to forecast streamflows (McLeod et al., 1977; Lu et al., 1996; Abrahart and See, 2000; Montanari et al., 2000; Ooms and Franses, 2001). Some nonlinear AR models are also applied to streamflow forecasting (Tong and Lim, 1980; Astatkie et al., 1997).

Several surveys of nonlinear time series models and monographs and texts written on the topic exist (e.g., Tong, 1990; Granger and Teräsvirta, 1993; Franses and van Dijk, 2000). There are shorter surveys highlighting different sections of the field (e.g., Brock and Potter, 1993; Teräsvirta et al., 1994; Potter, 1999; Swanson and Franses, 1999; Granger, 2001; Van Dijk et al., 2002; Tsay, 2002). This study is restricted to parametric models. For a

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recent treatment of nonparametric models, see Fan and Yao (2003). Deterministic processes are another area beyond the scope of considerations here (Tong, 1995).

Most streamflow processes are commonly accepted as nonlinear (Wang, 2006). Jian et al. (1998) fitted a mixed threshold autoregressive model to a river flow data set and reported that the proposed approach could accurately describe the complicated nonlinear time series and the discharge regime of the river flow. Vasas et al. (2007) analyzed a daily river flow using a two-state regime switching autoregressive model and stated that regime switching models have the advantage that they can be easily interpreted in physical terms with the latent regimes corresponding to wet and dry, or, alternatively, rising and falling periods. Astatkie (2006) compared forecasting performance of two nonlinear models with a nested threshold autoregressive (NeTAR) model using a daily streamflow, the results in this study suggested that the NeTAR model could be used for short term forecasting of daily streamflows of drainage basins with seasonal snow accumulation. Komornik et al. (2006) discussed the prediction ability of several nonlinear time series models including SETAR and STAR models and concluded that SETAR and STAR models could describe seasonal characteristics in the mean monthly streamflow data. Kisi (2009) investigated forecasting performance of the wavelet regression (WR) model in monthly streamflow forecasting, on the basis of the results, the WR is found to be better than the artificial neural network (ANN) and AR models in monthly streamflow forecasting. Shao et al. (2009) proposed functional-coefficient models with a periodic component for short-term streamflow forecasting, the proposed model extends many familiar nonlinear time series models including Exponential Autoregressive model (EXPAR) and Threshold Autoregressive model (TAR). Functional-coefficient models are capable of including exogenous variables such as upstream and rainfall in the model.

This study is in the context of operational forecasting and intended for forecasting daily flows. Our aim is to compare the forecasting performance of two linear models and five classes of nonlinear models, namely Threshold Autoregressive (TAR), Smooth Transition Autoregressive (STAR), Exponential Autoregressive (EXPAR), Bilinear Model (BL) and Markov Switching Autoregressive (MSAR) to capture the dynamics in the daily Colorado river discharge time series and illustrate performance of the models with respect to dry and wet seasons and flow magnitude. The Colorado river is a vital source of water for agricultural and urban areas in the southwestern desert lands of North America. The reasons for choosing these nonlinear models are: (1) in the river flow process beside well-recognized physical sources, some other sources such as seasonality, non stationarity and long memory can be identified as source of non linearity (Wang, 2006), regime switching models could describe seasonality and non stationarity of river flows; (2) the possibility of introducing simple non linear models to river flow forecasters; (3) it is well-known that there is a relationship between river flow and many meteorological variables, in particular rainfall–runoff process has asymmetric effects on river flow (Amendola et al., 2006), these effects can be adequately explained by different classes of regime switching models such as TAR, SETAR, and TARSO (e.g., Tong, 1990; Tsay, 1998); (4) switching models gives the possibility to model the short rising and longer falling periods separately, which is a distinguishing feature of the river flows (Vasas et al., 2007); (5) in fact, since precipitation data are rarely available for the whole river catchment, the use of latent regimes as proxies of rainfall is often the only way to incorporate the physical properties into the modeling process. We consider linear models since some authors argue that non linear models may fail to forecast better than simple linear models, even when linearity is rejected statistically (Clements and Krolzig, 1998; Teräsvirta et al., 2005).

We have conducted two verification methods, namely in-sample and out-of-sample forecasting (calibration and validation, respectively). In-sample forecasting (calibration) essentially tells us how the chosen model fits the data in a given sample while the out-of-sample forecasting (validation) is concerned with determining how a fitted model forecasts future values of the regressed, given the values of the regressors. For in-sample model comparison three criteria namely loglikelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC) are calculated. AIC and BIC criteria are objective measures of model suitability that balance model fit and model complexity in other word AIC and BIC are fitness indexes for trading off the complexity of a model against how well the model fits the data. Since AIC and BIC attempt to find the model that best explains the data with a minimum number of parameters, they are considered an approach favoring simplicity. The root-mean-square error (RMSE) and 9 other measures are used as the verification metrics to evaluate the out-of-sample forecast performance of linear and nonlinear models. These measures are the accuracy measuring criteria. The verification metrics are calculated using all data and data conditioned on seasons and flow magnitude. Least Squares (LS) and Maximum Likelihood (ML) methods are employed to estimate parameters of the models.

Sections 2–6 illustrate TAR, STAR, EXPAR, BL and MSAR models respectively, Section 7 introduces model comparison and forecast accuracy measures, Section 8 describes data, Section 9 presents empirical results, Section 10 explains out-of-sample forecasts and final section is devoted to the conclusion.

2. Threshold autoregressive

One popular class of nonlinear time series models is the Threshold Autoregressive (TAR) models, which is probably first proposed by Tong (1978) and discussed in detail in Tong (1990). The TAR models are simple and easy to understand, but rich enough to generate complex nonlinear dynamics. For example, it can be shown that the TAR models can have limit cycles and thus be used to model periodic time series, or produce asymmetries and jump phenomena that cannot be captured by a linear time series model. In spite of the simplicity of the TAR model form, there are many free parameters to estimate and variables to choose when building a TAR model, and this has hindered its early use. A special class of TAR model is called self-exciting TAR (SETAR).

2.1. TAR and SETAR models

Consider a simple AR(p) model for a time series $\{y_t\}$

$$y_t = \mu + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + \sigma\epsilon_t \tag{1}$$

where μ and α_i $(i = 1, 2, \ldots, p)$ are the AR coefficients, $\epsilon_t \sim N(0, 1)$ and $\sigma > 0$ is the standard deviation of the disturbance term. The model parameters $\alpha = (\mu, \alpha_1, \alpha_2, \ldots, \alpha_p)$ and σ are independent of time t and remain constant. To capture nonlinear dynamics, TAR models allow the model parameters to change according to the value of a weakly exogenous threshold variable z_t:

$$y_t = \mu^{(j)} + \alpha_1^{(j)} y_{t-1} + \alpha_2^{(j)} y_{t-2} + \cdots + \alpha_p^{(j)} y_{t-p} + \sigma^{(j)}\epsilon_t, \quad \text{if } r_{j-1} < z_t \leqslant r_j \tag{2}$$

where $j = 1, 2, \ldots, k$, and $-\infty = r_0 < r_1 < \cdots < r_k = \infty$. In essence, the $k-1$ non-trivial thresholds $(r_1, r_2, \ldots, r_{k-1})$ divide the domain of the threshold variable z_t into k different regimes. In each different regime, the time series y_t follows a different AR(p) model. When the threshold variable $z_t = y_{t-d}$, with the delay parameter d being a positive integer, the dynamics or regime of y_t is determined by its own lagged value y_{t-d} and the TAR model is called a self-exciting TAR or