



Semi-analytical and approximate solutions for contaminant transport from an injection well in a two-zone confined aquifer system



Ping-Feng Hsieh, Hund-Der Yeh*

Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan

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SUMMARY

This study develops a mathematical model for contaminant transport due to well injection in a radial two-zone confined aquifer system, which is composed of a wellbore skin zone and a formation zone. The model contains two transient equations describing the contaminant concentration distributions; one is for contaminant transport in the skin zone while the other is for transport in the formation zone. The contaminants are injected into the well with given dispersive and advective fluxes; therefore, the well boundary is treated as a third-type (Robin) condition. The solution of the model derived by the method of Laplace transforms can reduce to a single-zone solution in the absence of the skin zone. In addition, an approximate solution in the time domain is also developed by neglecting dispersion for the case that the contaminants move away from the injection well. Analysis of the semi-analytical solution showed that the influence of the skin zone on the concentration distribution decreases as time elapses. The distribution will be over-estimated near the wellbore if the constant concentration (Dirichlet) condition is adopted at the well boundary. The approximate solution has advantages of easy computing and yield reasonable predictions for Peclet numbers larger than 50, and thus is a practical extension to existing methods for designing aquifer remediation systems or performing risk assessments.

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1. Introduction

Radial contaminant transport problems have been intensively studied. Ogata (1958) was the first to develop an analytical solution using the complex integral method for radial transport problems with an injection of constant contaminant concentration at the wellbore, yet his solution was in terms of an integral form and cannot be evaluated numerically. Tang and Babu (1979) presented a complete solution in terms of Bessel functions and modified Bessel functions for radial transport; however, their solution was in a very complicated form and difficult to evaluate. Moench and Ogata (1981) solved the radial transport equation by Laplace transforms. They obtained the Laplace-domain solution involving Airy functions and then inverted the solution numerically using the Stehfest algorithm to the time domain. Later, Hsieh (1986) gave an analytical solution for the radial transport problem; this solution consisted of an integral form with Airy functions, and was computed by the 20-point Gaussian quadrature. Chen et al. (2002) applied a Laplace transform power series (LTPS) technique to solve the radial solute transport

equation with a spatially variable coefficient. The aquifer pumping induced a convergent flow field and the groundwater velocity therefore varied with radial distance. Their analytical results indicated that the LTPS technique can effectively and accurately handle the radial transport equation under the condition of high Peclet number. Later, Chen (2010) presented a mathematical model for describing three-dimensional transport of a contaminant originating from an area centered within a radial, non-uniform flow field. The solution of the model was developed by coupling the power series technique, the Laplace transform and the finite Fourier cosine transform. The comparison between this solution and the Laplace transform solution showed excellent agreement. Chen et al. (2012a) considered radial contaminant transport problems in a two-zone confined aquifer system and presented a semi-analytical solution for describing the concentration distribution in the aquifer system, which consists of a formation zone and a skin zone resulted from well drilling and/or well completion. Liu et al. (2013) derived a semi-analytical solution to the problem of groundwater contamination in an aquifer–aquitard–aquifer system, considering both advective transport and diffusive transport of contaminants in the aquifers and the intervening aquitard. All of the solutions mentioned above adopt a constant concentration condition at the inlet boundary, implying that the contaminants are well mixed and continuously enter the aquifer systems. In other words, those solutions were derived under the

* Corresponding author. Address: 300 Institute of Environmental Engineering, National Chiao Tung University, 1001 University Road, Hsinchu, Taiwan. Tel.: +886 3 5731910; fax: +886 3 5725958.

E-mail address: hdyeh@mail.nctu.edu.tw (H.-D. Yeh).

first-type boundary (or Dirichlet boundary) condition at the well boundary.

The third-type boundary condition, also called Robin boundary condition, which considers the effects of both the dispersive and advective fluxes, may also be adopted at the rim of the wellbore. This condition leads to conservation of mass inside the formation, while the wellbore has a well-mixed concentration at a constant flow rate entering the formation (Bear, 1972). Chen (1987) presented an analytical solution for radial dispersion problems with Robin conditions at the rim of the wellbore. The analysis of Yeh and Yeh (2007) indicates that the solution obtained from the contaminant transport equation with the Dirichlet boundary condition over-estimates the concentration near the wellbore if the flow regime is dispersion dominant. Pérez Guerrero and Skaggs (2010) presented a general analytical solution depicting solute transport with a distance-dependent dispersivity in a heterogeneous medium subject to a general boundary condition, which can be a first-, second-, or third-type. Veling (2012) presented a mathematical model describing the solute concentration distribution in a radial groundwater velocity field due to well extraction or injection. The model was composed of a radial transport equation with a Dirichlet, Neumann, or inhomogeneous mixed boundary condition at the well boundary. The solution of the model was solved using the methods of Laplace transform and generalized Hankel transform. Chen et al. (2011) developed an analytical model depicting two-dimensional radial transport in a finite-domain medium subject to either the first- or the third-type boundary condition at the well boundary. Recently, Chen et al. (2012b) derived a generalized analytical solution for the problem of coupled multi-species contaminant transport in a finite-domain medium under an arbitrary time-dependent third-type boundary condition. Wang and Zhan (2013) developed a mathematical model for describing radial reactive solute transport due to well injection in an aquifer-aquitard system consisting of a main aquifer and overlying and underlying aquitards. The well boundary was specified as either Dirichlet or Robin type. In fact, the Dirichlet boundary can be considered as a special case of the Robin boundary because the solution developed with the Robin boundary can reduce to the one with the Dirichlet boundary if the dispersion mechanism is negligible.

In the past, many studies have been devoted to the development of approximate solutions for radial dispersion contaminant transport problems. Raimondi et al. (1959) derived an approximate solution based on two assumptions. One was that the total derivative of the contaminant concentration with respect to time is equal to zero when the solute is far away from the well. The other was to neglect the radius of the injection well. Hoopes and Harleman (1967) presented a summary of earlier works and provided an approximate solution by neglecting the effect of a finite well radius. Later, both Dagan (1971) and Gelhar and Collins (1971) obtained approximate solutions by employing the perturbation method. Tang and Babu (1979) also presented an approximate solution based on the work of Raimondi et al. (1959) with the consideration of the well radius.

The objective of this study is to develop a mathematical model for describing radial contaminant transport in a two-zone confined aquifer with a Robin boundary condition specified at the injection well. The solution (i.e., in Laplace domain) of the model is derived by the method of Laplace transform, and the time-domain results (hereinafter referred to as “semi-analytical solution”) are obtained by the Crump algorithm (1976). In addition, an approximate solution in the time domain is also developed in terms of error and complementary error functions. The impacts of the skin zone and the use of different boundary conditions on the contaminant concentration distribution in the aquifer system are investigated based on the developed solution.

2. Methodology

2.1. Analytical solution

Some assumptions are made for the mathematical model describing radial transport of the injected contaminant in a two-zone confined aquifer system. They are: (1) the aquifer is homogeneous, isotropic and of uniform thickness, (2) the injection well has a finite radius and a finite thickness skin, and therefore the aquifer can be considered as a two-zone system, (3) the well fully penetrates the aquifer, and (4) the effect of molecular diffusion is negligible. The groundwater velocity in a steady-state radial flow system can be written as:

$$v = Q/2\pi rbn \quad (1)$$

where Q is a constant injection rate (L^3/T), r is a radial distance from the center of the wellbore (L), b is the aquifer thickness (L), and n is the aquifer porosity (-).

The governing equation describing the concentration distributions in the wellbore skin zone and formation zone are expressed, respectively, as:

$$\frac{\partial C_1}{\partial t} + v \frac{\partial C_1}{\partial r} = D_1 \frac{\partial^2 C_1}{\partial r^2} \quad \text{for } r_w < r \leq r_1 \quad \text{and } t > 0 \quad (2)$$

and

$$\frac{\partial C_2}{\partial t} + v \frac{\partial C_2}{\partial r} = D_2 \frac{\partial^2 C_2}{\partial r^2} \quad \text{for } r_1 < r < \infty \quad \text{and } t > 0 \quad (3)$$

where C_1 and C_2 are the contaminant concentrations in the skin zone (or called first zone) and formation zone (or called second zone) [M/L^3], respectively; D_1 and D_2 are the dispersion coefficients in the first zone and second zone defined by $D_1 = \alpha_1 v$ and $D_2 = \alpha_2 v$, respectively [L^2/T]; α_1 and α_2 are the radial dispersivities in the first zone and second zone, respectively [L]; t is the time since injection [T]; r_w is the well radius [L]; r_1 is the radial distance from the central of the well to the outer radius of the skin zone [L].

For the sake of convenience, the dimensionless forms for Eqs. (2) and (3) can be formulated, respectively, as:

$$\kappa \frac{\partial^2 G_1}{\partial \rho^2} - \frac{\partial G_1}{\partial \rho} = \rho \frac{\partial G_1}{\partial \tau} \quad \text{for } \rho_w < \rho \leq \rho_1 \quad \text{and } \tau > 0 \quad (4)$$

and

$$\frac{\partial^2 G_2}{\partial \rho^2} - \frac{\partial G_2}{\partial \rho} = \rho \frac{\partial G_2}{\partial \tau} \quad \text{for } \rho_1 < \rho < \infty \quad \text{and } \tau > 0 \quad (5)$$

where $G_1 = C_1/C_0$ and $G_2 = C_2/C_0$ are the dimensionless concentrations in the first and second zones, respectively; ρ is a dimensionless radial distance defined as $\rho = r/\alpha_2$ and the other two dimensionless radial distances are $\rho_w = r_w/\alpha_2$ and $\rho_1 = r_1/\alpha_2$; τ is a dimensionless time defined as $\tau = Qt/(2\pi bn\alpha_2^2)$; $\kappa = \alpha_1/\alpha_2$ is the ratio of the skin-zone dispersivity to the formation-zone dispersivity. Initially, the aquifer system is considered to be free from contamination; i.e., the contaminant concentrations in both the skin and formation zones are equal to zero and expressed as:

$$G_1(\rho, 0) = G_2(\rho, 0) = 0 \quad \text{for } \rho \geq \rho_w \quad (6)$$

The Robin condition specified at the wellbore boundary is expressed as

$$G_1(\rho_w, \tau) - \kappa \frac{\partial G_1(\rho_w, \tau)}{\partial \rho} = 1 \quad \text{for } \tau > 0 \quad (7)$$

The condition at the remote boundary is considered to be free from contamination and thus described as:

$$G_2(\infty, \tau) = 0 \quad \text{for } \tau > 0 \quad (8)$$

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