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High-order averaging method of hydraulic conductivity for accurate soil moisture modeling

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SUMMARY

Richards' equation (RE) is the most common mathematical expression for soil water movement in a porous medium. Despite advancements in numerical schemes and high-performance computing, the requirements of iterative computations and fine grids hinder further extension of the RE to multi-dimensional and large-scale applications. Averaging methods of hydraulic conductivity have been known to be one of the significant factors affecting the accuracy of numerical solutions of the RE, especially when coarse grids are used. In this study, we developed a high-order averaging method of hydraulic conductivity for accurate numerical modeling of the RE, which has a straightforward formula regardless of the soil conditions and produces high simulation accuracy when used on coarse grids. The developed method is based on the high-order upwind scheme, which is widely used for hyperbolic partial differential equations within a finite volume framework in order to prevent numerical oscillations near a discontinuity while preserving high-order accuracy. Numerical simulations of several one- and two-dimensional cases performed in the study indicate that the proposed method outperforms existing simple averaging methods and is also superior, or at least equivalent, to complex averaging methods over a wide range of soil textures, especially on coarse grids. In addition, the proposed method is straightforwardly extended to nonorthogonal grids by being combined with the coordinate transformation method and the extension is verified through multi-dimensional test cases as well as tests on a heterogeneous soil domain.

1. Introduction

Modeling of soil water movement is of great importance in various research areas, and significant advances have been made toward understanding soil moisture dynamics in the last decade (Bárdossy et al., 1995; Bronstert and Plate, 1997; Corradini et al., 2011, 1998; DiCarlo, 2013; among others). Soil moisture influences a range of environmental processes in a nonlinear manner, e.g. the partitioning of precipitation into runoff, infiltration, leakage, and evaporation; the mass and energy exchanges between soil and atmosphere; and the organization of natural ecosystems and biodiversity (Ridolfi et al., 2003; Vereecken et al., 2008; Western et al., 2002; and references therein). Richards' equation (RE) is the most common mathematical expression for soil water movement in a porous medium but does not have a general analytical solution. Despite important advances having been made in numerical solutions of the RE (An et al., 2012, 2011, 2010; Celia et al., 1990; Jones and Woodward, 2000; Kuráž et al., 2010; Lott et al., 2012), simulation of soil moisture using the RE still requires considerable computational effort, which hinders its extension to multi-dimensional and large-scale applications. Although relatively coarse grids are required to simulate a wide spatial domain for a long time period, previous studies have shown that simple conventional methods using coarse grids suffer from drawbacks of numerical instability and oscillation, resulting in large errors (e.g. Baker, 2006). For example, if the numerical accuracy obtained at a spatial resolution of 10 cm is available with a spatial resolution of 100 cm, we can simulate a 100-fold larger domain in two-dimensional space with the same numerical computation and also expand the available temporal length in a given time. However, complex and sensitive numerical conditions originating from heterogeneous and anisotropic features of the soil matrix compound the difficulties in employing coarse grids. Therefore, accurate numerical methods on relatively coarse grids are crucial for applying numerical solutions of the RE to real-world cases.

One of the significant factors affecting the accuracy of numerical solutions of the RE is the averaging method of hydraulic conductivity when coarse grids are used. Previous studies (Baker,

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1995; Belfort and Lehmann, 2005; Haverkamp and Vauclin, 1979; Szymkiewicz, 2009) have shown that simple averaging methods such as arithmetic, geometric, or upwind means may produce unacceptably large errors on coarse grids owing to the high nonlinearity of the hydraulic conductivity. Further, the arithmetic mean—one of the most popular averaging methods—often suffers from the drawback of numerical oscillations on coarse grids. In order to achieve accuracy within an acceptable level on coarse grids, the Darcian mean approach (Warrick, 1991) has been proposed. In this approach, the average hydraulic conductivity is estimated using Darcy's law by assuming a steady state between two adjacent nodes or cells. Subsequently, several methods based on the Darcian mean approach have been proposed (Baker, 2000, 1995; Gastó et al., 2002; Szymkiewicz, 2009). Most recently, Szymkiewicz (2009) proposed the Darcian-mean-type method, in which the flow is categorized into three types—infiltration, drainage, and capillary rise—and hydraulic conductivity is computed between the integrated mean and the upwind mean according to flow types. This method generally shows a good performance for a variety of soil textures and cases. However, the integrated mean, used for computation of the Darcian mean, incurs additional computational cost compared to the simple averaging method. Since the function of hydraulic conductivity is highly nonlinear and no general form exists to integrate the hydraulic conductivity function, numerical integration is usually required as an additional step. Belfort et al. (2013) showed that only the upwind mean can strictly preserve the monotonicity of the solution. They also proposed a simple switching method, in which the upwind mean is used when the arithmetic or geometric mean violates the monotonicity of the solution. Despite this method having a simple implementation and incurring lower additional computational cost, its accuracy should be determined between the arithmetic mean and the upwind mean. In addition, if the arithmetic or geometric mean violates the monotonicity in several steps, e.g. multi-dimensional cases, this method may become identical to that with the upwind mean.

The objective of this study is to develop and verify a new high-order method for averaging the hydraulic conductivity to obtain accurate numerical solutions of the RE in conjunction with coarse grids. The developed method is based on the high-order upwind scheme, which has been widely used since the 1980s for hyperbolic equations such as the compressible Euler equation (e.g. Chakravarthy and Osher, 1983; Harten et al., 1983; Roe, 1986; Sweby, 1984) or the shallow water equation (e.g. An and Yu, 2012; Louaked and Hanich, 1998; Mingham and Causon, 1998; Toro, 2000) for preventing numerical oscillations while capturing discontinuity accurately. In this study, the high-order upwind scheme is implemented for evaluating hydraulic conductivity. To the best of our knowledge, the high-order upwind scheme has not yet been applied to the evaluation of hydraulic conductivity in the numerical modeling of the RE, despite its high potential to improve the numerical accuracy on coarse grids. It is worth noting that the developed method is different from the conventional upwind mean, which has only first-order spatial accuracy.

The remainder of this paper is organized as follows. Section 2 presents the numerical scheme of the RE with averaging methods of hydraulic conductivity. It also describes several conventional averaging methods and the proposed method. Section 3 presents an evaluation of the proposed method through several test cases, including one-dimensional (1D) infiltration, drainage, and capillary rise problems; and two-dimensional (2D) infiltration and drainage problems. The 2D test cases consider the implementation of the proposed method on nonorthogonal grids and a heterogeneous soil domain. Lastly, Section 4 concludes the paper.

2. Numerical scheme

2.1. Numerical discretization of RE

The 1D form of the RE is written as follows (Richards, 1931):

$$\frac{\partial \theta(\psi)}{\partial t} - \frac{\partial q}{\partial z} = 0, \tag{1}$$

$$q = K(\psi)\frac{\partial}{\partial z}(\psi + z), \tag{2}$$

where ψ is the pressure head; θ, the volumetric soil moisture content; K, the hydraulic conductivity; t, the time; and z, the vertical dimension, which is assumed to be positive in the upward direction. As Eqs. (1) and (2) include both θ and ψ, this form of the RE is called the "mixed" form. This form is generally preferred to the θ-based and ψ-based forms for its improved mass balance in treating saturated and unsaturated flows simultaneously (Celia et al., 1990).

Applying implicit Euler temporal discretization and cell-centered finite-volume spatial discretization on a uniform grid to Eq. (1) gives

$$\frac{\theta_i^{m+1} - \theta_i^m}{\Delta t} - \left(\frac{K_{i+1/2}^{m+1}(\psi_{i+1}^{m+1} - \psi_i^{m+1})}{\Delta z^2} - \frac{K_{i-1/2}^{m+1}(\psi_i^{m+1} - \psi_{i-1}^{m+1})}{\Delta z^2} + \frac{K_{i+1/2}^{m+1} - K_{i-1/2}^{m+1}}{\Delta z} \right) = 0, \tag{3}$$

where the superscript m denotes the time level and the subscript i denotes the cell number. Note that fourth term on the left-hand side of Eq. (3) represents nonlinear advection originating from gravity force, and this term can cause numerical oscillation near the discontinuity unless an appropriate averaging method of hydraulic conductivity for $K_{i+1/2}^{m+1}$ and $K_{i-1/2}^{m+1}$ is implemented. Similarly, a multi-dimensional RE on an orthogonal grid can also be discretized within the cell-centered finite-difference or finite-volume framework, as shown in the previous studies (An et al., 2011; Clement et al., 1994).

A discretized equation is solved using the Newton iteration method. Since this method is usually sensitive to the initial guess, we implement a simple line-search approach to improve the robustness following Kelley (1995). The time-step duration is adjusted on the basis of the number of iterations required for convergence at the previous time-step duration (An et al., 2012; Paniconi and Putti, 1994). The time-step duration cannot be less than a preselected minimum or greater than a preselected maximum. If the number of iterations to convergence is less than N_m, the next time-step duration is multiplied by C_m, which is a predetermined value greater than 1. If the number of iterations is greater than N_r, the next time-step duration is multiplied by C_r, which is a preselected value less than 1. Further, if the number of iterations becomes greater than a prescribed N_b, the iterative process for the time level is terminated. Subsequently, the time-step duration is multiplied by C_b—a predetermined value less than 1—and the iterative process restarts. For most large and difficult problems, these control factors of the time-step duration will require adjustment to ensure good iterative performance. In this study, the values of $C_m = 1.2$, $C_r = 0.8$, $C_b = 0.5$, $N_m = 6$, $N_r = 10$, and $N_b = 20$ are used according to the previous study (An et al., 2012), where the parameters are optimized empirically.

As we deal with the RE in the "mixed" form, soil water retention and hydraulic conductivity models are required to express the relationships of θ–ψ and K–ψ, which impose difficulties in the numerical computation owing to high nonlinearity. Several mathematical models have been proposed in the literature (Szymkiewicz and Helmig, 2011; and references therein). In this study, because of a wide range of applications, we use two sets of soil water retention and hydraulic conductivity models: (1) the combination of the Brooks–Corey model (Brooks and Corey, 1964) and Mualem model