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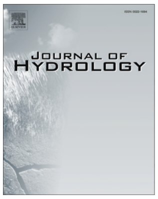

Stream filtration induced by pumping in a confined, unconfined or leaky aquifer bounded by two parallel streams or by a stream and an impervious stratum

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SUMMARY

A mathematical model is developed for describing three-dimensional groundwater flow induced by a fully-penetrating vertical well in aquifers between two parallel streams. A general equation is adopted to represent the top boundary condition which is applicable to either a confined, unconfined or leaky aquifer. The Robin (third-type) boundary condition is employed to represent the low-permeability streambeds. The Laplace-domain head solution of the model is derived by the double-integral and Laplace transforms. The Laplace-domain solution for a stream depletion rate (*SDR*) describing filtration from the streams is developed based on Darcy's law and the head solution and inverted to the time-domain result by the Crump method. In addition, the time-domain solution of *SDR* for the confined aquifer is developed analytically after taking the inverse Laplace transform and the time-domain solutions of *SDR* for the leaky and unconfined aquifers are developed using the Padé approximation. Both approximate solutions of *SDR* are expressed in terms of simple series and give fairly good match with the Laplace-domain *SDR* solution and measured data from a field experiment in New Zealand. The uncertainties in *SDR* predictions for the aquifers are assessed by performing the sensitivity analysis and Monte Carlo simulation. With the aid of the time-domain solutions, we have found that the effect of the vertical groundwater flow on the temporal *SDR* for a leaky aquifer is dominated by two lumped parameters: $\kappa = K_v x_0^2/(K_h D^2)$ and $\kappa' = K'D/(B'K_v)$ where D is the aquifer thickness, x_0 is a distance between the well and nearer stream, K_h and K_v are the aquifer horizontal and vertical hydraulic conductivities, respectively, and K' and B' are the aquitard hydraulic conductivity and thickness, respectively. When $\kappa < 10$, neglecting the vertical flow underestimates the *SDR*. When $\kappa \geqslant 10$, the effect of vertical flow is negligible. When $\kappa' \leqslant 10^{-4}$, the aquitard can be regarded as impermeable, and the leaky aquifer behaves as a confined one.

1. Introduction

Well pumping near a stream causes water filtration from the stream. The well could be vertical, horizontal, or vertical with laterals at the well bottom. The ratio of the filtration rate to the pumping rate is defined as a stream depletion rate (*SDR*). During the pumping period, *SDR* increases from zero to a constant value which could be equal to or less than unity. When *SDR* is zero, the filtration has not happened and the pumping has not affected the stream. *SDR* starts to increase with time when the drawdown cone reaches the stream. When *SDR* is unity, the stream filtration is at a rate which equals the pumping rate. The steady-state *SDR* is less than unity in the presence of additional recharge sources from such as an aquifer, stream, or/and long-term rainfall.

An analytical approach is commonly used to estimate *SDR* for problems involving stream water management and water rights. A variety of analytical and semi-analytical models associated with the prediction of temporal *SDR* have been proposed and categorized according to different aquifer types, well types and stream treatments. Most existing models consider a vertical well to fully penetrate an aquifer, implying that groundwater flow within the aquifer is two-dimensional (2-D). In addition, the stream is commonly treated as a boundary in the models or its effect is modeled as a source term in the governing equations of the models.

If a stream is assumed to fully penetrate an aquifer, it forms a boundary with the aquifer. Theis solution (1941) expressed in terms of the well function might be the first solution describing

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a temporal distribution of *SDR*. In the solution development, the stream is considered as the Dirichlet (first-type) boundary condition, and image well theory is used to develop the solution. Therefore, Theis solution (1941) did not account for the effect of the low-permeability streambed on temporal *SDR*. Glover and Balmer (1954) reduced Theis solution (1941) to a complementary error function for conciseness. Swamee et al. (2000) further developed a closed-form approximate expression for Theis solution (1941). Hantush (1965) treated a stream as the Robin (third-type) boundary and derived an analytical solution accounting for the low-permeability streambed effect. The solutions mentioned above neglect the effect of streambed storage on temporal *SDR*. Recently, Sun and Zhan (2007) derived an analytical solution considering two parallel constant-head streams and the effects of the streambed's storage and permeability. Intaraprasong and Zhan (2009) further derived an analytical solution with considering the effect of a variable stage stream.

A stream may be regarded as a source term in the governing equation of groundwater flow. The term is in terms of the Dirac delta function, implying that the stream has a zero width. Those solutions considering the source term are applicable to the groundwater problem in the presence of a low-permeability streambed. On the other hand, the source term represents a fully-penetrating stream effect with neglecting the vertical flow component (Sun and Zhan, 2007). Hunt solution (1999) might be the first analytical solution derived by treating the stream as a line source and was shown to be exactly the same as Hantush solution (1965) according to Sun and Zhan (2007). Chen and Yin (2004) extended Hunt solution (1999) by considering water exchange between a stream and an aquifer prior to pumping. Recently, Zlotnik and Tartakovsky (2008) treated a stream as a line source and presented an analytical solution for a leaky aquifer with leakage at the bottom of the aquifer.

In order to account for the effect of a stream width, some researchers divided an aquifer into three zones with different governing flow equations. The middle zone has a width equaling the stream width, and a source term is in its governing flow equation and distributed over the whole spatial domain. A fully-penetrating well is in one of the side zones and treated as a sink term in its governing equation. The other side zone considers no source or sink term in the governing flow equation. Those three governing equations are coupled via the continuities of head and flux at the interfaces between the middle zone and side zones (Zlotnik and Huang, 1999). Butler et al. (2001) used this approach to derive a semi-analytical solution for a confined aquifer and addressed the effect of the stream width on *SDR*. Fox et al. (2002) considered the same model as Butler et al. (2001) but derived an analytical solution in the time domain.

A multiple-layer aquifer system is commonly represented by a quasi three-dimensional (3-D) flow model in which the flow in the aquifer is horizontal and in the aquitard is vertical. The aquifer system may be classified into a leaky aquifer or two-layer aquifer system. The leaky aquifer consists of a main aquifer and an aquitard either on the top or at the bottom. The groundwater flow in the aquitard is assumed to be vertical due to the low hydraulic conductivity. Hunt (2003) developed an analytical solution for head and stream filtration in the leaky aquifer overlain by a thin aquitard with a free surface. The stream with a zero width is treated as a source term in the governing equation for the underlying aquifer. Hunt (2008) also considered the same aquifer but a finite width stream. The aquifer extends infinitely along the stream and is bounded by the no-flow boundaries in the direction perpendicular to the stream. He developed a semi-analytical solution for hydraulic head and stream filtration. Butler et al. (2007) derived a semi-analytical solution describing hydraulic head and stream filtration for the leaky aquifer with an underlying aquitard. The two-layer aquifer system has two main aquifers with an aquitard embedded in the middle. Hunt (2009) developed a semi-analytical solution for such an aquifer system. The stream is treated as a source term of a zero width, and a vertical well fully penetrates the upper aquifer. Recently, Ward and Lough (2011) considered the same situation but the well was installed at the lower aquifer. They derived a semi-analytical solution in the Fourier and Laplace domain for hydraulic head and in the Laplace domain for stream filtration.

Some researchers developed an analytical solution in predicting *SDR* induced by a slanted well, horizontal well or radial collector well. The solution takes account of the vertical component of groundwater flow even in a confined aquifer. Based on a 3-D groundwater flow equation, Tsou et al. (2010) derived an analytical solution for the temporal *SDR* induced by a slanted well in a confined aquifer. The slanted well can behave as a horizontal or vertical one by adjusting the orientation and inclination of the well. They found that the water flow filtration from a fully-penetrating stream toward a horizontal well parallel to the stream will reach steady state quickly. Huang et al. (2011) used a 3-D groundwater flow equation along with a simplified free surface equation representing the upper boundary of an unconfined aquifer and developed an analytical solution for the temporal *SDR* induced by a horizontal well. Their solution can investigate the effect of specific yield on *SDR*. These two solutions consider the stream as the Dirichlet boundary in the absence of a low-permeability streambed. Recently, Huang et al. (2012a) considered the streambed as the Robin boundary and presented an analytical solution to describe hydraulic head and temporal *SDR* induced by a radial collector well in an unconfined aquifer. They reported that the largest drawdown at the water table occurs right at the well center at the beginning of the pumping and moves landward when the filtration occurs.

Some semi-analytical solutions to a problem involving a horizontal well in a leaky aquifer underlying a water reservoir were also presented. The reservoir is of infinite extent in the horizontal direction and treated as a constant-head boundary at the top of the aquifer. Zhan and Park (2003) presented a semi-analytical solution for such a situation. The aquifer directly connects the overlying reservoir without a low-permeability aquitard in between. Sun and Zhan (2006) developed a semi-analytical solution for the same situation but considered the effects of aquitard storage and permeability.

Some researchers considered a wedge-shaped confined or unconfined aquifer and treated the adjacent stream as the Dirichlet boundary condition. Yeh et al. (2008) developed an analytical solution for the hydraulic head and *SDR* induced by a fully-penetrating vertical well in the wedge-shaped confined aquifer with an arbitrary angle. Singh (2009) also developed an analytical solution for *SDR* but for a right-angled confined aquifer. Sedghi et al. (2009) presented a semi-analytical solution for 3-D groundwater flow in the wedge-shaped confined or unconfined aquifer with a partially-penetrating vertical well. Other studies considered a triangle-shaped aquifer for simulating a delta aquifer surrounded by a stream. Asadi-Aghbolaghi and Seyyedian (2010) derived a closed-form solution describing a 2-D steady-state head distribution induced by a fully-penetrating vertical well in the triangle-shaped confined aquifer.

The solutions mentioned above are summarized in Tables 1 and 2. In Table 1, all of the solutions dealing with the 2-D flow problem induced by a fully-penetrating vertical well are categorized based on aquifer types and stream treatments. In Table 2, the solutions involving the quasi 3-D flow or 3-D flow are categorized based on aquifer categories, well types, and stream treatments. In these two tables, the superscripts *a* and *b* stand for the presentation of the time-domain solution and Laplace-domain solution, respectively.