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Modeling infiltration rates in a saturated/unsaturated soil under the free draining condition

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SUMMARY

We propose a model to simulate infiltration into heterogeneous soils, with arbitrary initial water content distributions, subject to unsteady rainfall, and under the free draining condition. It is a mass conservative theoretical approach based on the numerical resolution of the one-dimensional Richards equation. The model tracks the movement of the wetting front along the soil profile, checks the ponding status, and handles the shift between ponding and non-ponding conditions in the course of the simulation. It partitions the rainfall input into infiltration and surface runoff whenever the upper soil layer is saturated. We validate our theoretical approach by comparing the results for a homogeneous soil under steady rainfall with those calculated with CHEMFLO-2000. The differences between our approach and the modified Green-Ampt model are showed by considering the infiltration under unsteady rainfall conditions. Finally, we illustrate the model capabilities by examining the water flow and surface runoff for both homogeneous and stratified soils under intense rainfall events.

1. Introduction

The understanding of the water motion through soils is of great relevance for the appropriate use, management, and protection of our natural resources. It also has practical consequences in determining the adequate conditions to locate buildings of different types. Soil properties, as well as both initial and external conditions, may affect dramatically the direction and rate of the water movement. The development and resolution of theoretical models which account for those properties have a very positive impact in a number of disciplines, ranging from environmental science to architecture.

Numerous models have been proposed over the last decades to describe and predict water infiltration into soils. Many of them are based on a conceptual representation of the system, which inherently involves a number of important aproximations. Although they are very attractive for many applications because of their computational efficiency and stability, their ability to capture the daily dynamics is generally rather poor, especially for fine textured soils and thick profiles (Gandolfi et al., 2006). Progressively, more sophisticated theoretical approaches have been modified to take into account different factors, such as infiltration into heterogeneous soil profiles, unsteady rainfall events, water motion in different directions, or more realistic boundary conditions. In most cases, important approximations are still taken to obtain explicit expressions for the quantities of interest. For instance, the water motion in the modified Green-Ampt model (Chu and Marino, 2005) is assimilated to the vertical displacement of a piston, instantaneous hydraulic equilibrium is assumed at the interface between distinct soil layers, and the time-varying depth of ponded water at the ground surface is neglected. While these simplifications allow one to calculate analytically the relevant quantities, they may lead to significant deviations with respect to reality. Smith (1990) described general infiltration features of a two-layer system, including the crust case as a limit. Corradini et al. (2000) developed a conceptual model involving coupled ordinary differential equations for infiltration and soil water content distribution in a two-layered soil under any rainfall pattern, applicable in vertically homogeneous soils. Recently, this model was reformulated in a simpler approach for two-layered soil profiles with the upper layer much more permeable than the subsoil (Corradini et al., 2011b), and then extended to field-scale (Corradini et al., 2011a).

The Richards equation (Richards, 1931) establishes the liquid mass conservation through soils in gravity/pressure-driven flows. Although it is based on this fundamental principle, it commonly includes several significant approximations. Its simplest 1D version does not consider the flow in the horizontal directions, and

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does not include either source or sink terms. Richards' model relies on the Darcy-Buckingham constitutive equation for water movement in unsaturated soils, which may fail for swelling soils or preferential flow of water through large pores in contact with free water. In spite of all these limitations, the Richards model is amply recognized as the most appropiate framework in infiltration because accurately describes such a process in many practical situations. In fact, it has been frequently used as reference against which to validate more simplified approaches (Gandolfi et al., 2006; Assouline et al., 2007).

The exact solutions of the Richards equation are restricted to simplified descriptions of unsaturated hydraulic properties (Basha, 2011). Solutions for more realistic hydraulic models resort to important approximations that require further validation. Nevertheless, there is a considerable body of literature related to the search for analytical solutions of the Richards equation. Here, we just mention some examples. Parlange et al. (1999) represented the solution to the Richards equation by an expansion in powers of the depth retaining the linear and quadratic terms. Mollerup (2007) showed that a power series solution can be applied for variable-head infiltration, a description more appropriate than the constant-head condition for mostly natural occurring infiltration scenarios. A traveling wave exact solution of the boundary value problem for the Richards equation has been validated from comparison with numerical simulations (Zlotnik et al., 2007). Ghotbi et al. (2011) applied the so-called homotopy analysis method to solve the Richards model, finding accurate solutions. Assuming that the water content and permeability coefficient are exponential functions of the pressure head, and diffusivity is constant, Huang and Wu (2012) found analytical solutions to 1D horizontal/vertical water infiltration in saturated/unsaturated soils for a time-dependent rainfall.

The use of numerical techniques allows one to solve more sophisticated versions of the Richards model. CHEMFLO-2000 (Nofziger and Wu, 2012) is an interactive program designed to simulate the movement of water and chemicals in unsaturated soils under unsteady rainfall. For this purpose, a pressure-based form of the Richards equation is integrated with a finite difference scheme. The 2D Richards equation with a sink term have been solved numerically to simulate runoff in unsaturated soils (Bhardwaj and Kaushal, 2009). The numerical method employs a Galerkin finite element method for the spatial discretization and finite differences for the temporal one. This type of analysis has been combined with a wetting front-tracking approach to cope with both unsaturated and saturated flows (Browne et al., 2013). The numerical predictions were validated from comparison with experiments. Wu (2010) developed a finite element method to cope with the Richards equation. The comparison with the experiments showed that the numerical scheme provides satisfactory results over a wide variety of problems, such as infiltration fronts, steady-state and transient water tables, and transient seepage faces. The flow stability in infiltration processes can also be analyzed from the direct numerical simulation of the Richards model (Nieber et al., 2000). The traveling wave solution for gravity-driven flows becomes unstable owing to, e.g., water repellency, which leads to the appearance of fingering.

In this paper, we will present an infiltration model based on the numerical resolution of the Richards equation. The model is capable of simulating infiltration into non-uniform soils of arbitrary initial moisture distributions during unsteady rainfall. Contrarily to most numerical codes, our model switches from the unsaturated to the ponding condition and *vice versa* as the upper surface saturates and desaturates, respectively. This feature makes the algorithm very attractive to study infiltration into sloping soils under intense rainfall events, where the calculation of the surface runoff becomes a crucial aspect of the problem. Our model determines when ponding arises, and calculates the surface runoff in the course of the simulation. The model can be run to predict infiltration over long time periods with a reasonable computing time, and can be straightforwardly extended to analyze more complex geometrical configurations and soil compositions. We will validate the model and show the differences with respect to a simpler approach. More importantly, we will consider a real situation to illustrate the model capabilities. The surface runoff taking place over a long period of rainfall observations in Marbella (Spain) will be calculated for two types of soils. Finally, the flow through a stratified soil during an unsteady rainfall event will be analyzed.

Both the model and the numerical method are described in Section 2. The results are presented in Section 3. The paper closes with some conclusions in Section 4.

2. Mathematical model and numerical method

The flow through a vertical soil column of length L is calculated by solving the 1D Richards equation (Richards, 1931)

$$\frac{\partial\theta}{\partial t}+\frac{\partial q}{\partial z}=0 \tag{1}$$

for the water content $\theta(z,t)$. This quantity is the ratio of the water to soil volumes at the depth z and time t. In addition, $q(z,t)$ stands for the water flow rate per unit area crossing a horizontal surface. This rate is given by the Darcy-Buckingham law

$$q=K\left(1-\frac{d\psi}{dz}\right), \tag{2}$$

where $K(\theta)$ is the soil hydraulic conductivity, and $\psi(\theta)$ is the pressure head (measured in terms of height of a water column). Appropriate boundary conditions are to be imposed both at the bottom and top boundaries of the soil column. The free draining condition $\partial\theta/\partial z=0$ is assumed at the bottom end $z=L$, while a variable infiltration rate

$$I(t)\equiv q(0,t)=q_{\rm rain}(t), \tag{3}$$

is prescribed at the upper boundary $z=0$. This last condition only applies if $\theta(0,t)<\theta_s$. If the water content $\theta(0,t)$ reaches the saturation level θ_s, the boundary condition (3) is replaced with $\theta(0,t)=\theta_s$.

The problem formulated above is closed by considering the soil characteristic curves $K(\theta)$ and $\psi(\theta)$. A number of models (Brooks and Corey, 1964; Mualem, 1976; van Genuchten, 1980; Vogel and Cislerova, 1988) can be used to calculate those two functions. In this work, the hydraulic conductivity K and pressure head ψ are those originally given by Brooks and Corey (1964):

$$K(\theta)=K_s\phi^{\frac{2+3\lambda}{\lambda}},\quad \psi(\theta)=\psi_s\phi^{-1/\lambda}, \tag{4}$$

where $K_s=K(\theta_s)$ and $\psi_s=\psi(\theta_s)$ are the hydraulic conductivity and pressure head at saturation, respectively, while $\phi=(\theta-\theta_r)/(\theta_s-\theta_r)$ and θ_r are the effective saturation and residual water content, respectively. The parameters $K_s,\psi_s,\theta_s,\theta_r$, and λ must be determined experimentally for a given soil type. For stratified soils, they depend on the depth z.

In this work, Eqs. (1)–(4) are integrated numerically with a finite volume method (Versteeg and Malalasekera, 2007). The spatial discretization and boundary conditions are outlined in Fig. 1. The soil column is split into N slices of thickness $\Delta z=L/N$. The z values with (without) asterisks denote the depths of their centers (boundaries). The water content for the slice i is evaluated at its center, $\theta_i(t)=\theta(z_i^*,t)$, while the flow rate is calculated at its upper boundary, $q_i(t)=q(z_i,t)$. To avoid ambiguity in the calculation of the material properties of multilayer soils, the spatial discretization must be designed in such a way that the interface between