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Technical Note

Variability of stream flow discharge in response to self-similar random fields of temporal fluctuations in lateral inflow rate

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SUMMARY

This article presents the use of stochastic methodology for quantitative analysis of variability in stream flow discharge in response to fluctuations in lateral inflow rate, where the lateral inflow rate is considered to be the difference between rainfall and infiltration rates. In this work, we focus on the case where the temporal correlation structure of the fluctuations in the lateral inflow rate can be characterized by the statistics of random fractals. A closed-form expression quantifying the stream flow variability is therefore developed to investigate the influence of the fractal dimension of lateral inflow process and the size of time domain. It is found that the stream flow discharge variability increases with the time domain size, while the fractal dimension of lateral inflow process plays a role in the smoothness of fluctuations in stream flow discharge around the mean.

1. Introduction

Understanding and quantifying the conversion of rainfall–runoff process into stream flow discharge is one of the major tasks in water resources engineering, especially for a long-term management of available water resources. Temporal fluctuations in rainfall are generally recognized as being affected by a wide range of natural physical processes, the details of which cannot be anticipated precisely. Hence, there is a great deal of uncertainty associated with the quantification of surface lateral inflow to the stream along its reach as produced by the rainfall–runoff process. This prompted us to investigate how the temporal fluctuations in the lateral inflow rate influence the variability in the stream flow discharge.

Note that lateral inflow refers to any water added to the stream due to effluent seepage from ground water, overland flow, interflow or via small springs and seeps (e.g., Singh, 1995). This research is primarily concerned with the case that the source of lateral inflow is dominated by the rainfall. Therefore, the lateral inflow rate in this work is defined as the difference between rainfall and infiltration rates.

Rainfall events show significant variability on temporal scales. However, some observations indicate that the temporal distributions of fluctuations in rainfall fields do exhibit the properties of long-range correlation and scale invariance. These properties greatly simplify the statistical characterization of rainfall fields at time scales by using the concept of fractal objects (e.g., De Michele and Bernardara, 2005; Hubert et al., 1993; Menabde et al., 1997; Olsson et al., 1993; Schmitt et al., 1998; Venugopal and Foufoula-Georgiou, 1996; Veneziano et al., 1996). In other words, the temporal distribution of fluctuations in rainfall fields can be modeled according to self-similar random processes and their temporal correlation satisfies a power law (e.g., Hewett, 1986; Voss, 1985).

The surface lateral inflow to the stream is a direct consequence of the rainfall–runoff process. There is a need to address the uncertainty (variability) associated with the prediction of available stream water resources, which is the task undertaken herein. In the following analysis, the temporal fluctuations in the lateral inflow rate is considered to be self-similar random fields such that the temporal variability in the lateral inflow rate can be dealt with using a fractal description, where the lateral inflow rate represents the surface runoff mainly from rainfall.

In the following we present a stochastic analysis of one-dimensional transient stream flow subject to uniformly distributed lateral inflow along the side of the stream. The application of the perturbation-based nonstationary spectral techniques will lead to a closed-form solution for quantifying the variability in stream flow discharge. This solution provides a basis for assessing the impact of input parameters on the stream flow discharge variability.

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2. Problem formulation

Unsteady flow in a stream has traditionally been formulated based on the Saint–Venant system equations (e.g., Chow et al., 1988). In practical applications, the local and convective accelerations in the system equations are often neglected to simplify the analysis. The exclusion of accelerations thus leads the system equations to a single equation, known as the diffusion wave equation (e.g., Fan and Li, 2006; Gottardi and Venutelli, 2008; Moussa, 1996; Sivapalan et al., 1997; Sulis et al., 2010)

$$\frac{\partial Q}{\partial t} = D_h\left(\frac{\partial^2 Q}{\partial X^2} - \frac{\partial q_R}{\partial X}\right) - U\left(\frac{\partial Q}{\partial X} - q_R\right) \tag{1}$$

where Q is the stream flow discharge, D_h and U are the hydraulic diffusivity and wave celerity, respectively, and q_R is the lateral inflow rate (per unit stream length), which is considered to be uniformly distributed along the stream.

Eq. (1) is highly nonlinear due to the dependence of the diffusivity and celerity coefficients on the stream flow discharge Q, the dependent variable of (1). However, it may be linearized in a perturbation form based on the steady uniform reference values of the flow discharge and flow cross-sectional area written as (e.g., Lal, 2001; Moramarco et al., 1999; Yen and Tsai, 2001)

$$\frac{\partial Q'}{\partial t} = D_{h0}\frac{\partial^2 Q'}{\partial X^2} - U_0\left(\frac{\partial Q'}{\partial X} - q\right) \tag{2}$$

where $Q' = Q - Q_0$, $q = q_R - q_0$, and Q_0 and q_0 represent the steady uniform initial values. For a wide rectangular channel, for example, the diffusivity coefficient may be expressed in the form (e.g., Yen and Tsai, 2001)

$$D_{h0} = \frac{1}{2}\left[1 - \left(1 - 2\frac{U_0}{V_0} + \frac{U_0^2}{V_0^2}\right)F_0^2\right]\frac{V_0 Y_0}{S_0} \tag{3}$$

where V_0 and Y_0 are the uniform flow velocity and depth, respectively, S_0 is the channel bed slope, $F_0 = V_0/(gY_h)^{0.5}$, and Y_h is the hydraulic depth. The celerity coefficient is given by (e.g., Yen and Tsai, 2001)

$$U_0 = \frac{3}{2}V_0 \tag{4a}$$

using Chezy's formula and

$$U_0 = \frac{5}{3}V_0 \tag{4b}$$

using Manning's formula. Note that the term $\partial q/\partial X$ has been omitted from (2) due to the assumption of uniformly distributed recharge.

In the analysis presented below, the lateral inflow representing the source of stream flow is assumed to be a temporally correlated random field (a stochastic process based on the time series). It results in temporally correlated random fluctuations in stream flow discharge. That is, the stream flow discharge, the output (dependent variable) of the stream flow equation, is also treated as a random field. As such, the perturbation Eq. (2) provides a framework for quantifying the stream flow variability in terms of the temporal variability of the lateral inflow.

3. Stream flow variability analysis

We consider a weakly stationary random lateral inflow field in time so that the fluctuations in lateral inflow may be presented in form of Fourier-Stieltjes integral as:

$$q(t) = \int_{-\infty}^{\infty} e^{i\omega t} dZ_q(\omega) \tag{5}$$

where ω is the frequency and dZ_q is the complex random amplitude of the fluctuations. The perturbed quantity of stream flow discharge in (2) may be expressed by the Fourier-Stieltjes representation of a nonstationary process (e.g., Li and McLaughlin, 1991) as:

$$Q'(X,t) = \int_{-\infty}^{\infty} \Phi_{Qq}(X,t,\omega) dZ_q(\omega) \tag{6}$$

where $\Phi_{Qq}(X, t, \omega)$ is the transfer function.

Using (5) and (6), it follows from (2) that

$$\frac{\partial \Phi_{Qq}}{\partial t} = D_{h0}\frac{\partial^2 \Phi_{Qq}}{\partial X^2} - U_0\frac{\partial \Phi_{Qq}}{\partial X} + U_0 e^{i\omega t} \tag{7}$$

To gain a clear insight into the influence of the source variability on the stream flow, we focus only on the case where the boundary and initial conditions are deterministic (i.e., the case of lateral-inflow-dominated stream). Thus, the stochastic perturbation boundary and initial conditions associated with (7) take the forms:

$$\Phi_{Qq}(0,t) = 0 \tag{8a}$$

$$\Phi_{Qq}(L,t) = 0 \tag{8b}$$

$$\Phi_{Qq}(X,0) = 0 \tag{8c}$$

where L is the length of the stream. The system of Eqs. (7) and (8) admits the following solution:

$$\Phi_{Qq} = 2\pi U_0 \exp\left(\frac{\xi\mu}{2}\right)\sum_{n=1}^{\infty}\frac{n - n\exp\left(-\frac{\mu}{2}\right)\cos(n\pi)}{n^2\pi^2 + \frac{\mu^2}{4}}\sin(n\pi\xi) \times \frac{\exp(i\omega t) - \exp(-(\rho_1 n^2 + \rho_2)t)}{\rho_1 n^2 + \rho_2 + i\omega} \tag{9}$$

where $\mu = U_0 L/D_0$, $\xi = X/L$, $\rho_1 = D_0\pi^2/L^2$, $\rho_2 = U_0^2/(4D_0)$. A useful approximation may be made for the case of $\rho_1 t \gg 1$. For this case, the infinite series in (9) converges rapidly (e.g., Haberman, 1998) and (9) becomes

$$\Phi_{Qq} = 2\pi U_0\frac{1 + e^{-\mu/2}}{\pi^2 + \mu^2/4}e^{\mu\xi/2}\sin(\pi\xi)\frac{e^{i\varpi t} - e^{-\alpha t}}{\alpha + i\omega} \tag{10}$$

where $\alpha = \rho_1 + \rho_2 = (D_{h0}\pi^2 + \mu^2/4)/L^2$. In conjunction with (9), (6) is written as:

$$Q'(X,t) = 2\pi U_0\frac{1 + e^{-\mu/2}}{\pi^2 + \mu^2/4}e^{\mu\xi/2}\sin(\pi\xi)\int_{-\infty}^{\infty}\frac{e^{i\varpi t} - e^{-\alpha t}}{\alpha + i\omega}dZ_q(\omega) \tag{11}$$

Requiring from the representation theorem for Q', one obtains

$$\sigma_Q^2 = E[Q'Q'^*] = 8\pi^2 U_0^2\left(\frac{1 + e^{-\mu/2}}{\pi^2 + \mu^2/4}\right)^2 e^{\mu\xi}\sin^2(\pi\xi) \int_0^{\infty}\frac{1 + e^{-2\alpha t} - 2e^{-\alpha t}\cos(\omega t)}{\omega^2 + \alpha^2}S_{qq}(\omega)d\omega \tag{12}$$

where σ_Q^2 is the variance of the stream flow discharge, $E(-)$ stands for the ensemble average, the asterisk denotes the operation of complex conjugation, and $Sqq(\omega)$ is the spectrum of the lateral inflow perturbation.

As mentioned earlier, the temporal correlation structure of the rate of lateral inflow is assumed described by the statistics of random fractals. It has been demonstrated by Voss (1985) and Hewett (1986) that the spectral density of the fractal objects follows the power-law behavior. Hence, the spectrum of the lateral inflow $Sqq(\omega)$ in (12) has the form of

$$S_{qq}(\omega) = S_0/\omega^{\beta} \tag{13}$$

where S_0 is the spectral density at $\omega = 1$, β is the spectral exponent which can be related to the fractal dimension D. For one-dimensional fractal objects, $\beta = 5–2D$ and $1 < D < 2$. The reader is