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Mathematical model on grain-size distribution in suspension over sand-gravel bed

Debasish Pal *, Koeli Ghoshal

Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur 721302, India

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SUMMARY

In this study, the grain-size distribution in suspension over a sand-gravel bed in an open channel turbulent flow is investigated from theoretical point of view. On the basis of diffusion equation, a model on grain-size distribution is developed incorporating the effect of stratification and hindered settling due to increased suspension concentration. The hydrodynamic diffusion related to particle–particle interaction is considered in the computation of reference level and the influence of incipient motion probability, non-ceasing probability and pick-up probability of the sediment particles are considered in the computation of reference concentration. Due to inclusion of several factors, the proposed model predicts well the grain-size distribution of suspended load when compared with the experimental data and also shows its prediction superiority with respect to other existing models.

1. Introduction

In sediment transport study, non-cohesive sediments such as silt, sand or gravel start to move after exceeding the threshold shear stress and the transportation can be classified into two broad categories: bedload and suspended load. The bed load is the coarser fraction which move very closely to the bed by sliding, rolling and saltation and suspended load is the comparatively finer fraction of sediment particles which are in suspension for an appreciable period of time. In suspension, the grain-size distribution under different hydrodynamic conditions is a crucial topic of research due to its irregular behavior. Flume experiments had shown that the grain-size distribution in suspension is a function of bed materials, flow velocity and height of suspension and unimodality or log-normality of grain-sizes appears within a critical range of velocity and height above a sand bed of given composition (Sengupta, 1975, 1979; Sengupta et al., 1991). For computing the grain-size distribution in suspension, a mathematical model has been developed by Ghosh et al. (1981) where they first took into consideration the sorting process from the bed to bed layer and then sorting between the bed layer and suspension. In spite of predicting grain-size distribution in suspension directly, Ghosh et al. (1986) developed an interesting mathematical model which can be used to estimate the grain-size distribution of the deposited material by the grain-size distribution in suspension at higher velocity. On the basis of diffusion equation and taking into the effect of hindered settling, a mathematical model has been developed by Mazumder (1994) for computation of suspended grain-size distribution. Purkait (2002) studied the patterns of grain-size distribution of point-bar in the Usri River, India, and compared with log-normal, log-hyperbolic and log-skew-Laplace distribution models. His analysis revealed that the log-normal distribution is the best fitted distribution model for a particular size of bedforms. By the continuity equations of sediment and water, Mazumder et al. (2005b) developed a theoretical model for computation of suspended grain-size distribution by taking into the effect of suspension concentration into the mean velocity, viscous shear stress and turbulent shear stress. Ghoshal et al. (2011) statistically studied the grain-size distribution in suspension and their study showed that the grain-size distribution is leading from log-normal to log-skew-Laplace distribution with the increase of flow velocity and the bed roughness has no effect in changing the size distribution from log-normality. For predicting the grain-size distribution as well as total concentration in suspension, Pal and Ghoshal (2014) modified the Rouse equation by newly proposed empirical expressions of proportionality parameters β_i and β which are the ratios of sediment diffusion coefficient to the momentum diffusion coefficient for the ith grain-size class and total concentration respectively. The aforementioned studies mostly focus on the

* Corresponding author. Tel.: +91 7407989510.
E-mail address: bestdebasish@gmail.com (D. Pal).

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grain-size distribution in suspension over sand bed. Though an experimental study of grain-size distribution in suspension over a sand-gravel bed has been investigated by Ghoshal and Pal (2014) and the proportionality parameters present in the Rouse equation are modified on the basis of experimental data, still the study does not include many important effects like stratification and hindered settling. So it is worthy to develop model on grain-size distribution over sand-gravel bed including several effects of turbulence.

To develop a theoretical model of grain-size distribution in suspension over a sand-gravel bed, is one of the challenging tasks for its irregular behavior due to entrapment of the finer fractions of bed materials in the interstices of coarse fractions (Mazumder et al., 2005a). As a result, the particles available in the suspension are either greater or less than the expected quantity; consequently suspension prediction becomes difficult. Due to this difficulty, grain-size distribution in suspension over sand-gravel bed is still lacking in literature. For sand-gravel bed, the larger sediment sizes present in the bed are unlikely to go into suspension except under very high flow condition. But the smaller sediment sizes go easily in suspension and a stratification effect comes into play which is true for all kinds of sediment beds. According to available literatures, previous researchers considered the effect of stratification on total suspension concentration (Sheng and Villaret, 1989; Villaret and Trowbridge, 1991; Ghoshal and Mazumder, 2005; Herrmann and Madsen, 2007), but not on the grain-size distribution in suspension. Besides this, the previous researchers (Mazumder, 1994; Mazumder et al., 2005b) considered the effect of hindered settling by taking an integer value of reduction exponent of settling velocity in sediment-fluid mixture. But Cheng (1997a) had shown that the reduction exponent is a function of suspension concentration and particle diameter.

To overcome the limitations of inclusion of aforementioned effects, the main purpose of the present study is to develop a mathematical model for computing the grain-size distribution in suspension over a sand-gravel bed in an open channel flow taking into account the effects of stratification and the hindered settling due to increased suspension concentration where the reduction exponent is a function of concentration and particle diameter. Besides these effects, the influence of particle–particle interaction in also considered in the computation of reference level together with the effect of incipient motion probability, non-ceasing probability and pick-up probability of the sediment particle in the computation of reference concentration. The experimental data of grain-size distribution in suspension collected at Fluvial Mechanics Laboratory (FML) of Indian Statistical Institute (ISI), Kolkata, is used to examine the validity of the proposed model.

2. Mathematical model

For fully developed two dimensional open channel turbulent flow in which the origin of coordinates is on the bed (see Fig. 1), x-axis is along the streamwise direction and y-axis is perpendicular to x direction, the turbulent shear stress τ can be expressed by

$$\tau = -\rho\overline{u'v'} = \rho\epsilon_m\frac{\partial u}{\partial y} = \rho u_*^2\left(1-\frac{y}{h}\right) \quad (1)$$

where ρ is the mass density of fluid, u is the streamwise mean velocity, u' and v' are the fluctuation components of the streamwise and vertical flow velocity respectively, ϵ_m is the momentum diffusion coefficient, u_* is the shear velocity, y is the vertical distance from the sediment bed and h is the flow depth. The general form of eddy viscosity can be expressed by (Smith and McLean, 1977)

$$\epsilon_m = \epsilon_{m_n}(1-\alpha\beta R_F) \quad (2)$$

where ϵ_{m_n} is the neutral eddy viscosity that exists under neutral conditions, α is empirical constant, $\beta(=\epsilon_s/\epsilon_m)$ is the proportionality parameter in which ϵ_s is the sediment diffusion coefficient and R_F is the flux Richardson number which is defined as the ratio of sediment suspension energy to turbulent production in a sediment stratified flow. The expression of R_F can be written as

$$R_F = -g(s-1)\frac{\frac{\partial c}{\partial y}}{\left(\frac{\partial u}{\partial y}\right)^2} \quad (3)$$

where g is the gravitational acceleration, c is the volumetric suspension concentration of sediment particle, $s(=\rho_s/\rho)$ is the relative mass density of sediment particle in which ρ_s is the mass density of sediment particle. The adopted expression of ϵ_{m_n}, proposed by Nezu and Rodi (1986), is used in this study which is given by

$$\epsilon_{m_n} = \frac{\kappa u_* y(1-\frac{y}{h})}{1+\pi\Pi\frac{y}{h}\sin(\pi\frac{y}{h})} \quad (4)$$

where κ is the von Karman constant and Π is the wake parameter which is introduced by Coles (1956). Combining Eqs. (1), (2) and (4), one can get the expression of velocity gradient as

$$\frac{\partial u}{\partial y} = \frac{u_*}{\kappa y} + \frac{u_*\pi\Pi\sin(\pi\frac{y}{h})}{\kappa h} + \alpha\beta R_F\frac{\partial u}{\partial y} \quad (5)$$

Introducing the non-dimensional vertical height $\xi = y/h$, Eq. (5) can be rewritten as

$$\frac{\partial u}{\partial \xi} = \frac{u_*}{\kappa\xi} + \frac{u_*\pi\Pi\sin(\pi\xi)}{\kappa} + \alpha\beta R_F\frac{\partial u}{\partial \xi} \quad (6)$$

As the flow velocity for individual grain-size is not available in literature for grain-size distribution study, so for all grain-sizes it is assumed to be equal and as such some representative particle size of sediment bed is used. To obtain the velocity profile, all the parameters like settling velocity, reference level and reference concentration are calculated for this representative particle only. For a steady uniform sediment-laden flow, the rate of upward sediment flux $\epsilon_s\frac{\partial c}{\partial y}$ due to turbulence of the flow and the downward settling flux $\omega_R c$, ω_R being the settling velocity of representative sediment

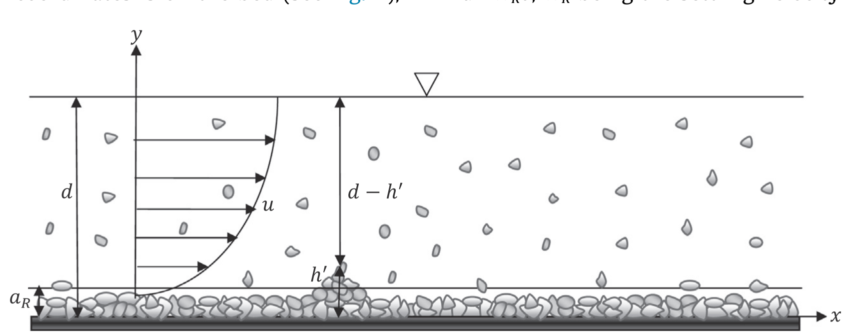

Fig. 1. The schematic diagram of turbulent flow over a non-uniform sediment bed through an open channel. Within the reference level a_R of representative grain-size, the sediments transport as bedload and above that as suspended load.