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Comparison of two model based approaches for areal rainfall estimation in urban hydrology

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SUMMARY

We introduce and compare two different approaches to estimate mean areal rainfall intensity in urban catchments. Both methods are based on the same lumped hydrological model that is calibrated beforehand. The first method uses a reverse model, i.e. an inverse formulation of a rainfall–runoff model. Rainfall intensities and their uncertainties are estimated from runoff data only. The second method estimates parameters of a rainfall error model using a Bayesian approach. It requires measurements of both runoff and rainfall. Although the two approaches are conceptually rather different, they address the same issue – the quantification of areal rainfall intensities and their related measurement errors – and a comparison is hence of interest. The merits and faults of the two methods are discussed. Results show that both methods provide best estimates of hyetographs with maximum intensities and total depths in a realistic order of magnitude, whereas the uncertainty of rainfall estimated with the reverse model is rather large.

1. Introduction – estimation of rainfall errors

Reliable information about rainfall intensity as the driving force of many relevant processes in urban catchments and drainage systems is of great importance. Rainfall is highly variable in space and time, and this variability is assumed to have a great influence on runoff quantity and quality at the outlet of a typical urban catchment. It should therefore be considered in simulation models (see e.g. Schellart et al., 2012; Schilling, 1984).

For many applications in urban drainage lumped conceptual models are used. These models are usually based on areal rainfall as model input, i.e. rainfall is assumed to be uniform in space. Rain gauges as common devices for rainfall observation perform point measurements, thus it is difficult to get information about the spatial variability and areal rainfall. Areal rainfall for a catchment can be estimated from several rain gauges by interpolation. However, as the rain gauge network is usually sparse with respect to the size of urban catchments, data from a single rain gauge is often assumed to represent areal precipitation and used as model input. Alternative methods such as rainfall radar (Einfalt et al., 2004; Marshall et al., 1947), precipitation estimates using satellite data (Kidd and Levizzani, 2011; Stephens and Kummerow, 2007) or microwave links (Messer et al., 2006) provide information about the spatial variability, but their accuracy is still limited as they estimate rainfall indirectly.

To improve model results, methods to estimate the rainfall error, i.e. the error of the rainfall data used as model input, have been developed. They range from simple approaches as areal reduction or correction factors (Vaes et al., 2005) to determine design storm intensities to very sophisticated methods based on dynamic error models, considering variation of the rainfall errors with time (e.g. Vrugt et al., 2008).

In this paper, we introduce and compare two model based methods to estimate rainfall with the purpose to determine the "true" areal rainfall and its uncertainties. The two methods estimate areal rainfall based on a hydrological model that is calibrated prior to rainfall estimation. They are conceptually rather different and have different data requirements. However, a comparison is interesting as they both address the important question of areal rainfall estimation.

The first method is based on a reverse model, i.e. a model to simulate rainfall based on measured runoff. The reverse model is an inverse formulation of a lumped rainfall–runoff model. Net

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areal rainfall intensities and their uncertainties are estimated based on runoff data and corresponding uncertainties by Monte Carlo (MC) simulation. No further model inputs are required. The second method uses a rainfall error model in addition to a rainfall–runoff model. Parameters of the error model representing rainfall measurement errors are estimated by Bayesian inference, i.e. a Markov-Chain Monte Carlo (MCMC) approach, based on measurements of both rainfall and runoff. Both methods are based on the same lumped conceptual model. However, whereas the reverse model estimates the rainfall using runoff data only, the error model considers also rain gauge data. The comparison of the two methods is performed by estimating areal rainfall in an urban catchment drained by storm sewers in Lyon, France.

Applications of reverse models to estimate rainfall in the context of urban drainage applications have been described by Marceau (1997) or Leonhardt et al. (2012). In these examples, equations of lumped rainfall–runoff models were rearranged to estimate areal rainfall from measured runoff. In both cases, the approach was used in the context of real time applications, i.e. rainfall intensities are estimated online based on runoff measurements from the past up to the current time. This implies some shortcomings, most important the required smoothing of runoff measurements. Another approach can be found in Hino (1986). As all rainfall intensities of an entire event are estimated simultaneously, it is limited to offline applications. He compared smoothed least squares and linear programming to estimate hourly rainfall in a natural catchment. However, physically meaningful results (i.e. positive rainfall) could only be obtained at the expense of strong smoothing. The method presented in this paper is also based on simultaneous estimation of the entire hyetograph using the least squares method. To ensure physically meaningful results an inequality constraint is imposed. Additionally, uncertainty in runoff measurements is considered to estimate rainfall uncertainties. The method is applied to high resolution runoff data from an urban catchment to estimate rainfall intensities in two-minute time steps.

Rainfall errors are considered as an important uncertainty source in hydrological models, but only recently, contributions are made aiming to address them explicitly in model calibration (e.g. Kavetski et al., 2006a,b; Reichert and Mieleitner, 2009; Renard et al., 2011). These studies incorporated rainfall uncertainty in hydrological model calibration based on the Bayesian total error analysis (BATEA) framework developed by Kavetski et al. (2006a,b). BATEA relies on a hierarchical Bayesian model to handle uncertainty in terms (e.g., rainfall) that are represented by error models using latent variables. The rainfall errors are computed in the calibration as other model parameters. The method presented in this paper also relies on a Bayesian approach, however, is not in the context of model calibration. In contrast to other applications that determine model parameters and rainfall errors together, this study is based on a pre-calibrated hydrological model and thus only rainfall errors need to be computed using the Bayesian method. To the best of the authors' knowledge, this is the first time rainfall error estimation is performed using the Bayesian method independent of model calibration.

Although this paper focuses on the introduction of the two methods, some possible applications should be mentioned. The estimated rainfall can be used as input to other lumped models, e.g. to simulate stormwater runoff quality. For those purposes, estimated areal rainfall might be more suitable than data from a single rain gauge. The estimated rainfall can furthermore be used to assess the quality or representativeness of the rain gauge data. The reverse model can also be used to fill rainfall data gaps. Furthermore, estimations of areal rainfall and its uncertainty might be of interest for the calibration and evaluation of indirect measurement methods. However, a data set of rainfall measurements is required for model calibration prior to application and rainfall estimates based on model approaches should not be interpreted independently of a model.

2. Methodology

The reverse model and the error model aiming to estimate areal rainfall, are based on the same hydrological model concepts. The conceptual hydrological model parameters are calibrated by a state of the art procedure, based on a calibration data set comprising rainfall and catchment runoff data. The reverse model is then applied to an evaluation data set. For the same data set, the rainfall error model is estimated and rainfall errors are computed.

This section describes the conceptual rainfall–runoff model as well as the methods used to estimate areal rainfall and its uncertainty, respectively, i.e. the reverse model and the error model. Model calibration is not the main scope of this paper as it is a pre-process for the application of the two methods. We therefore describe the calibration of the model parameters in the next section together with the case study catchment and the data.

2.1. Conceptual hydrological model

As already mentioned, a lumped conceptual hydrological model is considered in this study. The model consists of two parts: a rainfall loss component and a routing function. The gross rainfall is transformed to net rainfall after rainfall loss deduction. The net rainfall is then fed into the routing function to simulate runoff.

The rainfall loss incorporates an initial loss and a proportional loss. It is formulated as:

$$L(t) = \begin{cases} r & \text{if } \int_{t=0}^{t} r dt \leqslant L_0 \\ r p_{cons} & \text{if } \int_{t=0}^{t} r dt > L_0 \end{cases} \tag{1}$$

where $L(t)$ is the rainfall loss (mm/h), r (mm/h) is the rainfall intensity, L_0 and p_{cons} are the two model parameters representing the initial loss (mm) and the proportional loss (–). Net rainfall r_{net} can then simply be computed as follows:

$$r_{net}(t) = r(t) - L(t) \tag{2}$$

The routing function is represented by two cascaded linear reservoirs. The net rainfall r_{net} is converted to the inflow $q_{in,1}$ to the first reservoir with a lag time T_{lag} by multiplication with the impervious catchment area A:

$$q_{in,1}(t) = r_{net}(t - T_{lag})A \tag{3}$$

A linear reservoir has its outflow q_{out} varying linearly with its storage volume V:

$$q_{out}(t) = V(t)/K \tag{4}$$

where the parameter K (min) is called reservoir constant. Eq. (4) is combined with an equation of continuity:

$$q_{in}(t) - q_{out}(t) = dV(t)/dt \tag{5}$$

The analytical solution of Eq. (5) can be derived by integration of the differential equation over the time interval $[t - \Delta t, t]$:

$$q_{\mathrm{out}}(t) = \exp\left(-\frac{\Delta t}{K}\right) q_{\mathrm{out}}(t - \Delta t) + \left[1 - \exp\left(-\frac{\Delta t}{K}\right)\right] q_{\mathrm{in}}(t) \tag{6}$$

A second linear reservoir is placed in series after the first reservoir. The reservoir constants of the two cascaded reservoirs are set to the same value because they are heavily correlated when determining them in calibration according to some preliminary tests. A baseflow q_b to account for dry weather flow is simply added to the outflow from the second reservoir q_r as a contribution to the total