Journal of Hydrology 501 (2013) 45–55

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Finite volume approximation of the three-dimensional flow equation in axisymmetric, heterogeneous porous media based on local analytical solution

Amgad Salama [a,*], Wang Li [b], Shuyu Sun [a]

[a] King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Saudi Arabia
[b] Beijing Key Laboratory of Urban Oil and Gas Distribution Technology, China University of Petroleum, Beijing 102249, People's Republic of China

ARTICLE INFO

Article history:
Received 2 November 2012
Received in revised form 19 June 2013
Accepted 28 July 2013
Available online 6 August 2013
This manuscript was handled by Laurent Charlet, Editor-in-Chief, with the assistance of Chong-Yu Xu, Associate Editor

Keywords:
Flow in porous media
Cylindrical coordinate system
Analytical methods
Finite volume method

SUMMARY

In this work the problem of flow in three-dimensional, axisymmetric, heterogeneous porous medium domain is investigated numerically. For this system, it is natural to use cylindrical coordinate system, which is useful in describing phenomena that have some rotational symmetry about the longitudinal axis. This can happen in porous media, for example, in the vicinity of production/injection wells. The basic feature of this system is the fact that the flux component (volume flow rate per unit area) in the radial direction is changing because of the continuous change of the area. In this case, variables change rapidly closer to the axis of symmetry and this requires the mesh to be denser. In this work, we generalize a methodology that allows coarser mesh to be used and yet yields accurate results. This method is based on constructing local analytical solution in each cell in the radial direction and moves the derivatives in the other directions to the source term. A new expression for the harmonic mean of the hydraulic conductivity in the radial direction is developed. Apparently, this approach conforms to the analytical solution for uni-directional flows in radial direction in homogeneous porous media. For the case when the porous medium is heterogeneous or the boundary conditions is more complex, comparing with the mesh-independent solution, this approach requires only coarser mesh to arrive at this solution while the traditional methods require more denser mesh. Comparisons for different hydraulic conductivity scenarios and boundary conditions have also been introduced.

1. Introduction

Flow and transport in porous media are described, in the framework of the continuum hypothesis, using a set of partial differential equations provided certain conditions and length scale constraints are met, Salama and Van Geel (2008a,b). In this approach, field variables represent continuous functions of space and time that are differentiable to at least as much as the governing differential equations require. Although, in principal, these equations are simpler, in a sense, than those describing flow of fluids (e.g., the Navier–Stokes equations) they bear special characteristics that are unique to porous media. That is in the one hand, problems in porous media often span larger domain compared with that related to fluid flow applications. On the other hand, macroscopic properties of porous media are, in many situations, heterogeneous and sometimes are anisotropic compared with that in fluid systems which are usually isotropic. In the particular case of homogeneous porous media, macroscopic variables usually change in smooth gradual fashion which allows coarser mesh to be used when numerically solving the governing equations. There are, however, situations in which steep changes in variables may be encountered (e.g., in the vicinity of wells). In these cases, denser mesh would usually be required to obtain the mesh-independent solutions. That is, in the vicinity of the well region, pressure drops very fast between the far field value and that at the well. To capture this change, denser mesh is usually required, particularly when the medium is heterogeneous. This poses larger numerical loads particularly when the domain is large and highly heterogeneous. In particular, ground water management models require repeated simulation to identify an optimal solution to the management system (e.g., Sawyer et al., 1995, Ahlfeld and Hoque, 2008, Bayer et al., 2010, and many others). As an example, well optimization problems usually require large number of simulation exercises in order to identify the optimal location of production/injection wells. Therefore, it is important to search for numerical techniques that minimizes the simulation time (e.g., by allowing the use of coarser mesh and yet preserve the accuracy). Furthermore, the management of oil reservoirs has, in many cases, been dependent on accurate

* Corresponding author. Tel.: +966 8080423.
E-mail address: amgad.salama@kaust.edu.sa (A. Salama).

0022-1694/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jhydrol.2013.07.036

Nomenclature

A, B	constants defined in Eq. (11)
$\mathbf{k}$	hydraulic conductivity tensor
$\mathbf{g}$	gravitational vector
N	number of grid points
$\mathbf{v}$	velocity
$\mathbf{r}$	position vector
q	source
r	radius
p	local pressure
r, θ, Z	cylindrical coordinates
ρ	fluid density

Subscripts

P, N, S, E, W	center of the cell as well as at the north, South, East, and West cells
n, s, e, w	at the faces of the cell
T, B	top and bottom cells
t, b	top and bottom faces of the cell

estimation of well parameters (e.g., flow rates, well pore pressure, etc.) which apparently are sensitive to the accuracy of well region models. Therefore the problem of flow in the vicinity of wells has been the subject of intensive research because of its practical importance, Sparipalli et al. (2000). One can find two approaches to model the near well region; the first one is derived by the fact that since the length scale associated with the well dimension is much smaller than typical grid blocks, it may be possible to simulate the well as a source/sink term in the governing equations. In this case it is important to introduce interaction terms which relate well bore pressure, well block pressure, and well flow rate to close the system of equations, Peaceman (1978, 1983). Unfortunately, these interaction terms are not easily measured and turn out to be fitting parameters. Furthermore, near well regions are characterized by steep changes in pressure and that flow pattern is radially dominating which is not well-captured in reservoir flow models, Mundal et al. (2010). The second approach has been through modeling the well as part of the simulation domain. In this case no interaction terms are needed and realistic coupling between well region and the porous medium domain is achieved. However, as explained earlier, finer mesh will generally be needed in the vicinity of the well to capture the steep change in the pressure field which is, indeed, computationally expensive. In groundwater applications, aquifers are characterized as confined or unconfined whereas petroleum reservoirs are always confined. The basic difference between confined and unconfined aquifers is that, when pumped, confined aquifers are not dewatered. As pointed out by Louwyck et al. (2012), there exist models in literatures to treat radially-dominated flows towards wells that are either specific to certain scenarios and boundary conditions using analytical expressions (e.g., Butler, 1998; Kruseman and de Ridder, 1990; Reed, 1980) or more generic including semi-analytical or numerical solutions (e.g. Hemker and Maas, 1987; Veling and Maas, 2009; Pandit and Aoun, 1994; Reilly, 1984; Bohling and Butler, 2001; Johnson et al., 2001, etc.). With respect to petroleum reservoirs,Pedrosa and Aziz, 1986, introduced a near-well logarithmic discretization scheme in which an orthogonal curvilinear grid (cylindrical or elliptical) in well regions is used. In this approach a geometrical factor is introduced to the calculations of transmissibility. On the other hand Ding and Jeannin (2001, 2004) and Mundal et al. (2010), introduced the multipoint flux approximation to handle anisotropic near well region.

In this work we introduce a finite volume approximation to the problem of flow in well region. We introduce a cell-wise analytical expression to the flow problem in the radial direction, which allows for the use of coarser mesh and therefore appreciably reduces the simulation time. This technique works for three-dimensional, heterogeneous porous media encompassing an injection/production well, considering orthogonal curvilinear grid.

In the framework of finite volume method, the flux components are defined at the center of the faces defining the boundaries of the control volume, Versteeg and Malalasekera (1995) and Date (2005). This requires an accurate representation of the hydraulic conductivity at such cell faces. There are three approaches to define the hydraulic conductivity at the interface, these are: the arithmetic mean, the harmonic mean or the integral mean interpolation methods. Arithmetic mean represents a linear interpolation between two control nodes, which makes it easy to evaluate. However, more rigorous analysis suggests that harmonic mean should be used instead of the arithmetic mean to evaluate the hydraulic resistances at the interface boundaries. On the other hand, in the context of heat conduction, Voller and Swaminathan (1993) presented an integral mean interpolation scheme based on Kirchhoff transformation. They show that their scheme provides higher accuracy than arithmetic and harmonic means. However, it is more involved and usually requires numerical integration during the calculations, which is usually cumbersome, particularly when the integral of diffusion coefficient cannot be expressed analytically. The major assumption of the harmonic average of the conductivity at cell interfaces is the continuity of the flux calculated using forward difference, backward difference or center difference. That is the traditional way to calculating the harmonic mean is valid when the flux vector does not significantly change along its direction. Apparently, this is not the case in radially-dominated flows where the flux vector changes appreciably along its direction. In order to minimize this effect, denser mesh is required so that the change in the flux along its direction is not significant. In this work we generalize the technique developed by Li et al. (2011) that is based on local analytical solution to the problem of the flow in three-dimensional, heterogeneous and axisymmetric porous medium domain. A new formula for the harmonic mean calculations at cell faces normal to the direction of the radius is also introduced. Moreover, it should be mentioned that this formulation can be extended to multiphase flow in porous media including CO_2 sequestration, capillary driven two-phase flow, Cai and Yu (2011), etc. This technique has the advantage that it enables the use of coarser mesh and yet yields accurate results.

2. Governing equations

Consider a porous media domain, Ω, bounded with the boundary, $\partial\Omega$, the governing flow equations for this system may be described as:

$$\mathbf{v} = -\mathbf{k}\nabla(p - \rho\mathbf{g}\cdot\mathbf{r}) \quad \text{in } \Omega \tag{1}$$

$$\nabla\cdot\mathbf{v} = q \quad \text{in } \Omega \tag{2}$$

$$p = p_B \quad \text{on } \partial\Omega_p \tag{3}$$

$$\mathbf{v}\cdot\mathbf{n} = v_B \quad \text{on } \partial\Omega_v \tag{4}$$