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Approximate analytical solution for non-Darcian flow toward a partially penetrating well in a confined aquifer

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SUMMARY

In this study, non-Darcian flow to a partially penetrating well in a confined aquifer was investigated. The flow in the horizontal direction was assumed to be non-Darcian, while the flow in the vertical direction was assumed to be Darcian. The Izbash equation was employed to describe the non-Darcian flow in the horizontal direction of the aquifer. We used a linearization procedure to approximate the non-linear term in the governing equation enabling the mathematical model to be solved using a combination of Laplace and Fourier cosine transforms. Approximate analytical solutions for the drawdown were obtained and the impacts of different parameters on the drawdown were analyzed. The results indicated that a larger power index n in the Izbash equation leads to a larger drawdown at early times, while a larger n results in a smaller drawdown at late times. The drawdowns along the vertical direction z are symmetric if the well screen is located in the center of the aquifer, and the drawdown at the center of the aquifer is the largest along the vertical direction for this case. The length of the well screen w has little impact on the drawdown at early times, while a larger length of the well screen results in a smaller drawdown at late times. The drawdown increases with K_r at early times, while it decreases as K_r increases at late times, in which K_r is the apparent radial hydraulic conductivity. A sensitivity analysis of the parameters, i.e., the specific storage S_s, w, n and K_r, indicated that the drawdown is not sensitive to them at early times, while it is very sensitive to these parameters at late times especially to the power index n.

1. Introduction

The well-known Darcy's law, which states a linear relationship between the specific discharge and hydraulic gradient, has been used successfully to solve a series of groundwater problems. However, Darcy's law may become invalid if the flow velocity is relatively small or large (Engelund, 1953; Basak, 1977; Sen, 1985). It is generally accepted that when the flow velocity is greater or smaller than a certain critical value, flow becomes non-Darcian (Wilkinson, 1955; Slepicka, 1961; Dudgeon, 1964). Any deviations of the linear relationship between the specific discharge and the hydraulic gradient can be regarded as non-Darcian flow. The investigations on hydraulics of flow to a pumping well are very important in several areas including hydrogeology, environmental engineering and petroleum engineering (Cai et al., 2010). Most well hydraulic problems assume Darcian flow (i.e. Darcy's Law applies). However, flow can be non-Darcian near the pumping wells because of the high velocities. Using Darcy's law to describe such non-Darcian flow leads to an underestimation of the drawdown in the production well (see for example Mathias and Todman, 2010). Therefore, a series of empirical equations have been obtained to describe the relationship between the specific discharge and the hydraulic gradient for non-Darcian flow (Forchheimer, 1901; Izbash, 1931; Muskat, 1937; Rose, 1951; Escande, 1953). These equations can be classified into two general types, i.e., polynomial functions and power functions. The first type assumes that hydraulic gradient is a polynomial function of the specific discharge, while the second type assumes that the hydraulic gradient can be described as a power function of the specific discharge. The most popular equations of these two types are the Forchheimer equation (Forchheimer, 1901) and the Izbash equation (Izbash, 1931).

Many attempts have been made to derive analytical solutions to describe non-Darcy flow towards pumping wells using both the Forchheimer equation and the Izbash equation. Some earlier work sought to apply the Boltzmann transform was used to reduce the partial differential equation to a self-similar ordinary differential equation. In particular, Sen (1988a,b) applied such an approach to a Darcian concentric aquifer problem, and then to non-Darcy problems using the Izbash equation (Sen, 1989) and then the Forchheimer equation (Sen, 1990). However, it has been subsequently demonstrated that none of these three problems are self-similar in this respect (Mathias et al., 2008) and leads to considerable errors when wellbore storage is considered (Wen et al., 2009).

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Wen et al. (2008a,b,c) proposed an alternative linearization procedure to solve such non-Darcian flow problem to a pumping well providing an accurate approximation for non-Darcy problems during the late stage of pumping. However, the linearization procedure is based on the assumption of the flow rate at any cylindrical cross section is equal to the pumping rate Q. Obviously, this assumption only becomes valid after sufficient time has passed for the system to develop a quasi steady state. Thus, this linearization procedure can lead to substantial errors during the early stage of pumping. Typically, this can result in an underestimation of drawdown at early times (Wen et al., 2008b).

As mentioned above, one cannot obtain the exact analytical solution with the Boltzmann transform and the linearization procedure. Meanwhile, numerical solutions for non-Darcian flow to a pumping well have also been developed. For instance, Wu (2002) solved non-Darcian flow in a fractured reservoir with the Forchheimer equation using finite-differences. Mathias et al. (2008) also used the finite difference method to solve non-Darcian flow to a pumping well in a confined aquifer with the assumption that the non-Darcian flow can be described by the Forchheimer equation. Wen et al. (2009) investigated a similar problem with the finite difference method based on the assumption that the non-Darcian flow can be described by the Izbash equation.

A careful check of the recent literatures, one can find most of research on non-Darcian flow to a pumping well are focused on the fully penetrating well (Yeh and Chang, 2013). However, it is often desirable to simulate a partially penetrating well (denoted as PPW in the following). The main difference between the hydraulics of a PPW and a fully penetrating well is that the development of vertical flow around the well-bore. Vertical flow for fully penetrating wells can generally be neglected when dealing with confined aquifers. However, vertical flow can play an important role on the hydraulics of a PPW even in confined aquifers. Thus, one should consider both the horizontal flow and vertical flow when investigating PPW problems.

One of the first mathematical models for the Darcy flow to a PPW was presented by Hantush (1957). He obtained an analytical solution using the Laplace transform for time and Fourier transform for vertical distance. Following this work many variations have been presented for flow towards a PPW (e.g., Zlotnik et al., 1998; Luther and Haitjema, 1999; Chien and Chen, 2002; Yang and Yeh, 2005; Chiu et al., 2007; Chang and Yeh, 2009; Chen et al., 2010; Ataie-Ashtiani et al., 2012). For instance, Luther and Haitjema (1999) investigated steady state flow to one or more partially penetrating wells in an unconfined aquifer and an analytical element solution was obtained. Chiu et al. (2007) also obtained an analytical solution of the drawdown for flow toward a PPW with considering a finite-thickness skin effect. Ataie-Ashtiani et al. (2012) derived an analytical solution for the capture zone of a PPW with skin effects on the basis of the well radius is infinitesimally small under a constant pumping rate in a confined aquifer.

However, it is noticed that all above mentioned studies on the flow to a PPW are based on assumption of Darcian flow. Actually, non-Darcian flow is also likely to occur near a PPW.

In this study, we investigate the effects of non-Darcian flow to a PPW in a confined aquifer. For tractability, flow in the horizontal direction is assumed to be non-Darcian, while the flow in the vertical direction was assumed to be Darcian. The Izbash equation was employed to describe the non-Darcian flow in the main aquifer. We used a linearization procedure, similar to previously proposed by Wen et al. (2008a,b,c), to approximate the non-linear governing equation. Then the mathematical problem is solved using a combination of Laplace transform and cosine Fourier transform. Approximate analytical solutions for the drawdown are then obtained and presented to explore the impact of different parameters on the drawdown.

2. Problem statement and its analytical solutions

2.1. Problem statement

In contrast to the case of a fully penetrating well, for partially penetrating wells (PPW), the flow velocity in the vertical direction cannot be neglected. Because of the high velocity in the horizontal direction near the pumping well, non-Darcian flow is likely to occur, especially when the pumping rate is relatively large. In this study, we assume that the flow near the pumping well is non-Darcian, and accords to the Izhash equation.

The following additional assumptions are invoked: (1) the aquifer is confined, homogenous, and horizontally isotropic; (2) the pumping well partially penetrates the aquifer, and the well radius is infinitesimally small; (3) the discharge of pumping well is constant and equal to Q; (4) the aquifer is infinite in the horizontal direction; (5) the whole system is hydrostatic before the pumping starts; and (6) the horizontal specific discharge q_r is assumed to be constant along the screen. A schematic diagram of the studied system is shown in Fig. 1.

Based on the assumptions above, the mathematical model can be established as follows:

$$\frac{\partial q_r}{\partial r}+\frac{q_r}{r}+\frac{\partial q_z}{\partial z}=S_s\frac{\partial s(r,z,t)}{\partial t}, \tag{1}$$

$$s(r,z,0)=0 \tag{2}$$

$$s(\infty,z,t)=0 \tag{3}$$

$$\frac{\partial s(r,0,t)}{\partial z}=0 \tag{4}$$

$$\frac{\partial s(r,M,t)}{\partial z}=0 \tag{5}$$

$$\lim_{r\to 0} rq_r=-\frac{Q}{2\pi w}[U(z-d)-U(z-l)], 0\leqslant z\leqslant B \tag{6}$$

where r[m] is the distance to the center of the pumping well and t[h] is the pumping time; z[m] is the vertical coordinate; s[m] is the drawdown; q_r[m/h] is the horizontal specific discharge; q_z[m/h] is the vertical specific discharge; S_s[m^{-1}] is the specific storage of aquifer; Q[m^3/h] is the pumping rate; B[m] is the thickness of aquifer; l[m] is the distance from the top of the well screen to the bottom of the aquifer; d[m] is the distance from the bottom of the well screen to the bottom of the aquifer; w[m] is the length of well screen which is defined as $w = l-d$; $U(\cdot)$ is the Heaviside step function, $U(z-d)$ is zero when z less than d, otherwise, $U(z-d)$ equals one.

Horizontal specific discharge is described by the Izbash equation:

$$q_r|q_r|^{n-1}=K_r\frac{\partial s(r,z,t)}{\partial r} \tag{7}$$

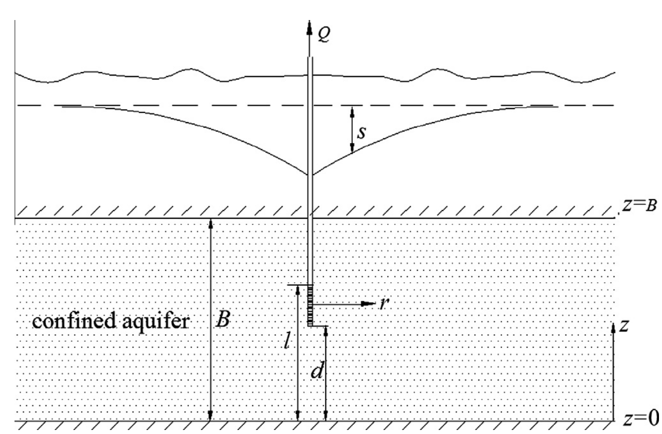

Fig. 1. The schematic diagram of the flow system.