Journal of Hydrology 499 (2013) 200–209

Contents lists available at SciVerse ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Modeling contaminant transport in a two-aquifer system with an intervening aquitard

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ARTICLE INFO

Article history:
Received 26 January 2013
Received in revised form 1 June 2013
Accepted 26 June 2013
Available online 10 July 2013
This manuscript was handled by Peter K. Kitanidis, Editor-in-Chief, with the assistance of Todd C. Rasmussen, Associate Editor

Keywords:
Analytical solution
Finite difference solution
Aquitard
Laplace transform
de Hoog et al.'s method
Tauberian theorem

SUMMARY

This study deals with the issue of one-dimensional solute transport in a two-aquifer system, where an aquitard lies between two aquifers. Different from previous studies on analysis of the contaminant transport affected by the presence of an aquitard, we developed a mathematical transport model in an aquifer–aquitard–aquifer system with considering transport of solutes in the aquitard governed by both advection and diffusion. The Laplace-domain solution of the model for concentration distributions is obtained by the Laplace transform technique and its corresponding time-domain results are computed numerically by using Laplace numerical inversion. An explicit finite difference model is also developed to simulate two-dimensional contaminant transport process in the system. The simulated depth-averaged concentrations in the lower and upper aquifers slightly differ from those predicted by the present solution. The results show that the movement of contaminant in the upper aquifer is slowed down considerably due to the advective transport in aquitard. When neglecting the aquitard advection (a zero Peclet number), the concentration level in the lower aquifer will be underestimated, especially at late times. In addition, the contaminant concentration in the lower aquifer increases significantly with aquitard's Peclet number.

1. Introduction

Groundwater contamination from deliberate disposal or accidental spill of chemicals in aquifers has received much concern for the quality of water resources. It is rather complicated to analyze or predict the migration of contaminants in layered geologic formations analytically. Many aquifers with stratigraphic features are bounded above and/or below by low permeable layers, referred to as aquitards. Previous studies have demonstrated that the aquitard plays an important role in the behavior of subsurface flow and the migration of hazardous materials from underground storage tanks or industrial waste landfill (e.g., Johnson et al., 1989; Parker et al., 2004). The effect of the presence of an aquitard on the migration of contaminants is, however, commonly neglected or handled based on some simplifications from previous studies on flow and transport in stratigraphic formations. When the solutes migrate in an aquifer–aquitard–aquifer system, the solutes may penetrate the aquitard due to molecular diffusion. Furthermore, advective flux of solutes may also penetrate the aquitard due to the presence of hydraulic gradient produced by pumping in the adjacent aquifer (Cherry et al., 2006, p. 11) and/or other driving force such as concentration or temperature gradients (Freeze and Cherry, 1979, p. 25) between the aquifers. Thus, transport by advection in the aquitard might also be a significant transport process in the aquifer–aquitard–aquifer system and that desires consideration. Zhan et al. (2009a) mentioned that the advective flux in the aquitard should be considered in modeling contaminant transport if the aquitard is thin. For a thin aquitard, the solute may penetrate the aquitard and enter the adjacent aquifer. As such, it is of importance to include the contaminant transport in the adjacent aquifer in modeling contaminant transport in multilayered aquifer systems.

In the past, many studies had been devoted to analyze the effect of aquitards on groundwater flow systems. For instance, Hantush and Jacob (1955) assumed that the confined aquifer is bounded from below and above by aquitards of finite vertical extent in which flow is entirely vertical and the effect of the aquitard's elastic storage is negligible. In addition, the flow in the confined aquifer is essentially horizontal. From verification of the assumption of vertical flow in aquitards and horizontal flow in the aquifer, Neuman and Witherspoon (1969) concluded that "When the permeabilities of the aquifers are two or more orders of magnitude greater than that of the aquitard, errors introduced by this assumption are usually less than 5%". Zlotnik and Zhan (2005) and Hunt and Scott (2007) investigated the aquitard effect on the results of pumping tests by assuming that

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0022-1694/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jhydrol.2013.06.050

a large conductivity contrast exists between the pumped aquifer and the aquitard, implying that the flow is horizontal in the pumped aquifer and vertical in the aquitard.

Diffusion at an aquifer–aquitard interface is somewhat similar to diffusion at a matrix-fracture boundary. It has been shown that matrix diffusion is an important process for contaminant transport in a fractured medium (Tang et al., 1981; Liu and Yeh, 2003; Liu et al., 2004). As a consequence, the advective flux in the aquitard has been neglected deliberately in previous analyses to make the derivation of analytical solutions tractable. Parker et al. (2004) showed that the migration of dissolved contaminants in low-permeability materials is typically dominated by molecular diffusion and may occur over time periods of hundreds to thousands years. Johns and Roberts (1991) proposed a model for investigating solute transport in large-aperture fractures under the consideration of lateral dispersion to the small aperture regions and diffusion to the rock matrix. Liu and Ball (2002) and Chapman and Parker (2005) recognized that back diffusion of the solute from the aquitard to the aquifer is the primary cause of the tailing effect observed in the aquifer. Note, however, that the solute transport by advection in aquitards was not taken into account in the above-mentioned studies. Zhan et al. (2009b) demonstrated that the mass transported between the aquifer and aquitard is sensitive to the aquitard's Peclet number, but less sensitive to the aquitard's diffusion coefficient, particularly at late times. In addition to the diffusive flux, their results implied that the advective flux in the aquitard is an important transport process for the contaminant to penetrate through the aquitard.

The objective of this paper is to develop a new mathematical model to describe contaminant transport in an aquifer–aquitard–aquifer system. Different from previous studies, this model considers the migration of contaminants by both advection and diffusion processes in the aquitard. The solution of the model in the Laplace domain is developed using Laplace transforms with the aid of both Ferrari's solution and Cardan's solution (Korn and Korn, 2000) and its corresponding results in the time domain are computed by de Hoog et al.'s algorithm (1982). The steady-state solution is also obtained from the Laplace-domain solution through the use of Tauberian theorem (Yeh and Wang, 2007). The concentrations predicted from this new solution are compared with the simulated depth-averaged concentrations from a two-dimensional explicit finite-difference model. Those newly developed solutions quantify the contaminant transport in an aquifer–aquitard–aquifer system and can be used to analyze the influences of aquitard properties on contaminant transport.

2. Conceptual and mathematical model

2.1. Conceptual model

Many aquitards exhibit variations in thickness or major internal lithology and therefore are often discontinuous in geologic facies at the regional scale. Cherry et al. (2006) presented a series of conceptual models for aquitards due to variations in depositional settings and post-depositional processes. Fig. 1 shows the schematic representation of the problem investigated in this study. The origin is located at the lower left-hand corner of the upper aquifer. The arrow shows the groundwater flow direction in both aquifers. Advection and diffusion are the physical processes controlling the transport of contaminants from the upper aquifer to the lower one through the aquitard.

2.2. Mathematical model

The assumptions related to the geometry and hydraulic properties of an aquifer–aquitard–aquifer system in the conceptual model are made as follows:

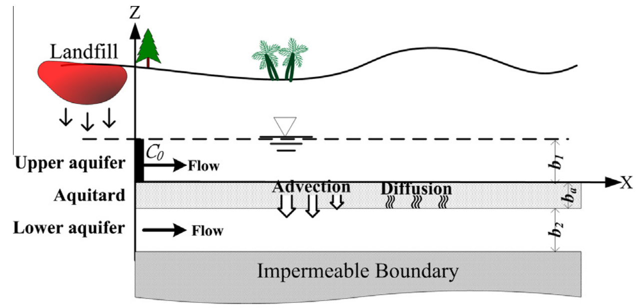

Fig. 1. Schematic representation of two-aquifer systems.

1. The flow and transport in both aquifers and aquitard is one dimensional. In addition, the flow fields are steady and uniform in the both aquifers and aquitard.
2. The hydraulic conductivity of the aquitard is a few orders of magnitude less than those of two adjacent aquifers, thus the direction of advective flow in the aquitard is vertical, i.e., perpendicular to the interface.
3. The aquifers and aquitard are homogeneous and isotropic with constant dispersivities and retardation factor.
4. The contaminants with a concentration kept constant at the inlet enter the two-aquifer system through the left boundary of the upper aquifer, while the lower one is initially not contaminated.

Based on these assumptions, the governing equations and associated initial and boundary conditions for the upper aquifer, aquitard, and lower aquifer are given below:

For the upper aquifer,

$$\frac{\partial C_1}{\partial t} = \frac{D_1}{R_1}\frac{\partial^2 C_1}{\partial x^2} - \frac{v_1}{R_1}\frac{\partial C_1}{\partial x} - \frac{\Gamma_1}{\theta_1 b R_1} \tag{1}$$

$$\Gamma_1 = \theta_a\left(v_a C_a - D_a\frac{\partial C_a}{\partial z}\right)\Bigg|_{z=0} \tag{2}$$

$$C_1(x,0) = 0 \tag{3}$$

$$C_1(0,t) = C_0 \tag{4}$$

$$C_1(\infty,t) = 0 \tag{5}$$

where C_1 and C_a represent the contaminant concentration in the upper aquifer and the aquitard, respectively; C_0 is the constant concentration at $x = 0$; v_1 is the average horizontal velocities of groundwater flow in the upper aquifer; v_a is the vertical velocity of groundwater flow in the aquitard; D_1 is the longitudinal dispersion coefficients for the upper aquifer and defined as $D_1 = \alpha_L v_1 + D^*$ with the longitudinal dispersivity α_L and the molecular diffusion coefficient in water D^*; D_a is the diffusion coefficient for the aquitard and defined as $D_a = \tau D^*$ with aquitard tortuosity τ; variable b is the half thickness of the aquifers; θ_1 and θ_a are the porosities of the upper aquifer and the aquitard, respectively; R_1 is the retardation factor in the upper aquifer, and t is elapsed time.

For the aquitard,

$$\frac{\partial C_a}{\partial t} = \frac{D_a}{R_a}\frac{\partial^2 C_a}{\partial z^2} - \frac{v_a}{R_a}\frac{\partial C_a}{\partial z} \tag{6}$$

$$C_a(x,z,0) = 0 \tag{7}$$

$$C_a(x,0,t) = C_1(x,t) \tag{8}$$

$$C_a(x,-b_a,t) = C_2(x,t) \tag{9}$$