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Redistribution of groundwater evapotranspiration and water table around a well field in an unconfined aquifer: A simplified analytical model

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SUMMARY

Groundwater evapotranspiration (GE) is an essential component for regional water balance in areas with shallow water table. When the depth to water table (DWT) falls due to pumping in a well field, GE will decrease, which also influence the redistribution of water table. A mathematical model of steady radial flow is developed to investigate the problem. The well field is simplified by a circular area with uniformly distributed pumpage. Relationship between GE and DWT is described with a commonly used linear formula when DWT is less than an extinction depth. Analytical solutions for three cases with or without a zero-GE zone (no GE occurs in the zone) are obtained. The redistributions of GE and water table are controlled by three lumped parameters. The analytical model is applied in a case study of the Ordos well field in China, with 55 pumping wells. It sufficiently agrees with a numerical model based on MODFLOW except in the vicinity of individual wells where cones of water table falling appear. The result of GE is not influenced by the discrepancy in water table due to existing of the zero-GE zone. Runge–Kutta solutions of using nonlinear GE formulas are presented and also applied in the case study to compare with the analytical model. They predict almost the same water table distribution and similar redistribution patterns of GE. The analytical model can be applied to determine the protection zone of well fields located in places without strong heterogeneous in topography and initial water table.

1. Introduction

Groundwater evapotranspiration (GE) is the uptake of moisture from underground water table to land surface caused by soil evaporation and plant transpiration. GE is an essential process in water circulation and influences the regional water balance in many aquifer systems. Previous theoretical analyses (Philip, 1957), laboratory studies (Gardner and Fireman, 1958) and in situ observations (Grismer and Gates, 1988) showed that GE decreases with increasing depth to water table (DWT). The decline of water table due to climate change or human activities can deteriorate or destroy land surface ecosystem, especially in arid and semiarid regions (Stromberg et al., 1996; Cooper et al., 2006), because it is more difficult for plants to uptake water for a greater DWT. Thus, before pumping of groundwater in an unconfined aquifer, we should be aware of the possible negative impacts on the environment. Accordingly, prediction of the redistributions of both water table and groundwater evapotranspiration around well fields is important.

In analyzing the interactions between water table drawdown and GE during pumping, a well flow model is required. The theory of radial flow towards wells in confined and unconfined aquifers, which is a major component of groundwater hydraulics, provides fundamental knowledge for the topic. The first well flow model was presented by Dupuit (1863), which deals with the steady state shape of water table in a circular unconfined aquifer that is bounded by surface water and pumped by a well at the center of the aquifer. A number of more robust models and equations have been developed in the last 150 years for steady or unsteady radial flow driven by a pumping well or a group of wells (Theis, 1935; Jacob, 1946; Hantush and Jacob, 1955; Hantush, 1956, 1960, 1964, 1967; Boulton, 1963; Neuman, 1972; Moench, 1984; Wang et al., 2004). In these contributions, the impacts of various aquifer conditions and configurations on groundwater flow have been investigated. These include aquifer setting (confined, unconfined, semi-confined, leaky aquifer, fractured aquifer), hydraulic condition around the wellbore (constant pumping rate or constant head), well screen configuration (fully or partially penetrated, infinitesimal or finite well radius, with and without wellbore

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Nomenclature

B	dimensionless parameter, Eq. (34)
d	depth to water table, DWT [L]
d_m	the extinction depth [L]
d_r	a reference depth, Eq. (47) [L]
d_t	the transition depth, Eq. (47) [L]
E	the rate of groundwater evapotranspiration (GE) [LT^{-1}]
E_m	maximum GE rate [LT^{-1}]
h	the height of water table above the aquifer bottom [L]
h_C	the dimensionless critical height of water table, Eq. (10)
h_D	the dimensionless height of water table, Eq. (10)
h_0	initial height of water table [L]
h_t	dimensionless parameter, Eq. (52)
I	infiltration rate [LT^{-1}]
k	hydraulic conductivity [LT^{-1}]
Q	pumping rate [L^3T^{-1}]
q	sink term, Eq. (2) [LT^{-1}]
r	radial distance from the center of the well field [L]
R	the radius of a well field [L]
r_c	the radius of the zero-GE zone [L]
r_D	dimensionless radial distance
r_w	the radius of a pumping well [L]
s	water table drawdown [L]
S_y	specific yield (−)
t	time [T]
t_D	dimensionless time, Eq. (44)
z_0	elevation of ground surface relates to the aquifer bottom [L]
α	dimensionless pumping rate, Eq. (11)
β	dimensionless parameter, Eq. (11)
ε	dimensionless sink/source term, Eq. (20)
λ	similar to β, defined in Eq. (49)

storage, well skin effect) and even the relation between water table and the capillary zone. One can see an intensive review on these advances in well hydraulics in Yeh and Chang (2013). However, only a few of the models considered the impact of GE on water table during pumping. Motz (1978) firstly developed a coupled aquifer model with both water table drawdown and GE reduction in the upper unconfined aquifer that influenced by a pumping well fully penetrated into the lower confined aquifer. The explicit solutions in Motz (1978) are available for steady state flow. Further, Denis and Motz (1998) extended the model for transient flow driven by pumping wells in both of the two aquifers. These previous analytical studies were carried out under the assumption of a constant decreasing rate of GE subject to per unit of water-table drawdown. This assumption is unrealistic because the relationship between groundwater evapotranspiration and water table is usually more complicated than that. When GE is considered in groundwater models, for example in MODFLOW-96 (Harbaugh and McDonald, 1996), an extinction depth is generally included in the GE formulas to handle the fact that GE rate would approximate zero if DWT is sufficiently great. Ignoring this relationship between DWT and GE may cause significant errors in evaluation of groundwater flow near pumping wells.

Redistributions of water table and GE around a well field can be investigated with numerical or analytical methods. Numerical models approximately represent the characteristics of flow in aquifers with specified physical based parameters, rather than represent general behaviors. In contrast, analytical solutions can reveal how groundwater flow and change in GE will be generally controlled by parameters that linked with each other. Exact explicit equations of analytical models are simple for application and can adopt variable conditions with similar processes. In this study, an analytical model is developed to find such exact explicit equations in describing the steady unconfined flow towards a well field with variable GE. The well field is simplified by a circular area with uniformly distributed pumpage. Relationship between GE and DWT is described with a linear formula when DWT is less than an extinction depth. Analytical solutions are obtained for three situations with low, medium and heavy pumping rates. The solutions represent general redistribution patterns of GE and water table around well fields: a zone of zero GE rate will appears with an area larger than (heavy pumping rate) or smaller than (medium pumping rate) the well field or be absent with a low pumping rate. In a case study, this analytical model is applied to investigate a well field with 55 pumping wells and compared with a numerical model dealing with individual wells. Runge–Kutta solutions of using nonlinear GE formulas are also presented and compared with the analytical solution in the case study. Limitations of the analytical model are discussed as well.

2. Mathematical model

2.1. Assumptions and equations

In this study, the following simplifications are applied and some of them are similar to those of well flow models in the literature:

(1) The aquifer is unconfined, homogenous and isotropic, and extends infinitely.
(2) The initial water table is horizontal with a height h_0 [L] above the datum, which is set as the impermeable bottom of the aquifer. GE rate with respect to the initial DWT is balanced by the infiltration rate. The infiltration recharge is assumed to be constant and uniformly distributed.
(3) Pumping wells are fully penetrated into the aquifer, and uniformly distributed in the well field with radius of R [L], as shown in Fig. 1. Each well has the same constant pumping rate and the total pumping rate of the wells is Q [L^3T^{-1}].
(4) The Dupuit assumption is assumed applicable so that the vertical flow in the aquifer is neglected.

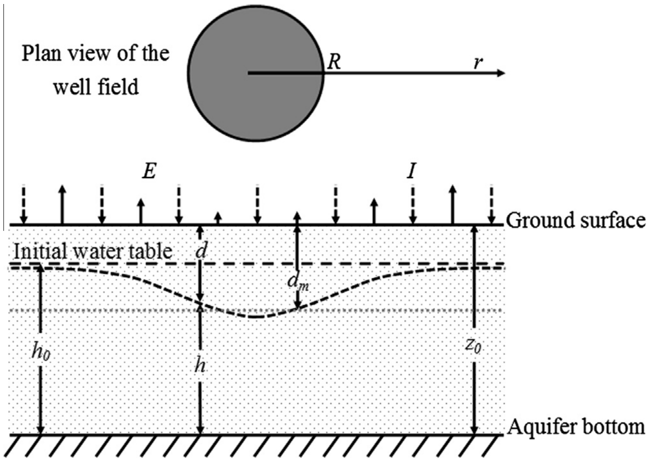

Fig. 1. Schematic representation of the well field (upper) and the aquifer conditions (lower). R is the radius of the well field. h_0 Is the initial height of water table. I and E represent the infiltration recharge and evapotranspiration discharge of groundwater, respectively.