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An effective inversion strategy for fractal–multifractal encoding of a storm in Boston

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SUMMARY

Hydrologic data sets such as precipitation records typically feature complex geometries that are difficult to represent as a whole using classical stochastic methods. In recent years, we have developed variants of a deterministic procedure, the fractal–multifractal (FM) method, whose patterns share not only key statistical properties of natural records but also the fine details and textures present on individual data sets. This work presents our latest efforts at encoding a celebrated rainfall data set from Boston and shows how a modified particle swarm optimization (PSO) procedure yields compelling solutions to the inverse problem for such a set. As our FM fits differ from the actual data set by less than 2% in maximum cumulative deviations and yield compression ratios ranging from 76:1 to 228:1, our models can be considered, for all practical purposes, faithful and parsimonious deterministic representations of the storm.

1. Introduction

Modeling of rainfall complexity has witnessed substantial progress in the past few decades, largely owing to the development of sophisticated mathematical techniques, such as those based on stochastic theories and fractal geometry. Although these ideas have resulted in a new language for the description and simulation of some of the data sets' intricacies, oftentimes these notions are still inadequate to study, on an individual basis, the incredible variety of natural rainfall patterns available to us.

Given that rainfall sets are typically erratic, noisy, intermittent, complex, or in short, "seemingly random," it has become natural to model them using stochastic (fractal) theories (e.g., Rodríguez-Iturbe, 1986; Lovejoy and Schertzer, 1990). This has inspired a variety of approaches that, while yielding realizations that preserve relevant statistical (physical) attributes of the records (e.g., moments, autocorrelation, power spectrum, multifractal spectrum, etc.), fail to capture specific details (e.g., positions of major peaks) and relevant textures (e.g., periods of no activity) present in measured data sets.

These limitations, intrinsic to any stochastic approach, led us to develop a fractal geometric methodology (e.g., Puente, 1996) aimed at capturing the complexity of rainfall patterns, and not just some key statistical features. By interpreting data sets as deterministic derived measures obtained transforming multifractals via fractal interpolating functions (e.g., Barnsley, 1988), our "fractal–multifractal" (FM) approach can indeed generate a vast class of patterns, over one or more dimensions, that encompasses all the distinctive characteristics of rainfall sets (e.g., Puente, 1996; Obregón et al., 2002a, 2002b) and other complicated geophysical patterns such as contaminant plumes in heterogeneous geological formations (e.g., Puente, 2004). This richness in the possibility of generating complex-looking deterministic sets with a relatively small number of parameters results, however, in a very intricate structure of the associated parameter space. Owing to this complexity, the solution of the inverse problem for a give data set, that is, searching for suitable FM parameters that produce a "match," remained, to date, an elusive task.

In this article, we report on our latest efforts to solve this very involved inverse problem. The proposed inversion strategy builds upon a recent generalization of the classical particle swarm optimization (PSO) search procedure (Fernández Martínez et al., 2010) that, combined with a statistical sampling of the initial conditions for the PSO, yields near-perfect parameter recovery for synthetic data sets and excellent fits of natural historic rainfall

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records, such as a detailed Boston storm used in earlier studies (Rodríguez-Iturbe et al., 1989; Puente and Obregón, 1996; Obregón et al., 2002b).

2. Materials and methods

In this section, we summarize the underlying mathematics of our FM geometric construction, including some of the extensions we have introduced, and outline the strategy for the solution of the inverse problem.

2.1. The original FM approach

In its simplest and original form, a FM pattern is obtained as the projection of the graph of a fractal interpolating function illuminated by a multifractal measure, as follows.

Firstly, the graph $G = \{(x, f(x)) | x \in [0, 1]\}$ of such a fractal function $f{:}x \to y$ passing by $N + 1$ ordered points along x, $\{(x_n, y_n) | x_0 < \ldots < x_N, n = 0, 1, \ldots, N\}$, is defined as the unique deterministic attractor of N simple affine maps: (Barnsley, 1988)

$$w_n\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \end{pmatrix}, \quad n = 1, \ldots, N, \tag{1}$$

where the vertical scaling parameters d_n satisfy $|d_n| < 1$, and the other parameters a_n, c_n, e_n, and f_n are defined via the contracting initial conditions

$$w_n\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_{n-1} \\ y_{n-1} \end{pmatrix}, \tag{2}$$

$$w_n\begin{pmatrix} x_N \\ y_N \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \end{pmatrix}, \tag{3}$$

which map the end values of the data in x into internal subintervals.

In a practical setting, the graph of a fractal function f, typically shaped as a convoluted wire and having a fractal dimension $1 \leqslant D < 2$, is obtained by a pointwise sampling of the attractor via iterations of the affine maps, a procedure also known as the chaos game (Barnsley, 1988). The idea is to start the process at a point already in G, e.g., a given (x_n, y_n), and progressively iterate the N maps w_n according to, for example, the outcomes of independent "coin" tosses.

Secondly, as the chaos game is performed for a sufficient amount of time, not only is the set G found, but also a unique invariant measure is induced over G, which reflects how the attractor is filled up. The existence of such a measure (akin to a histogram) allows computing unique—and hence, fully deterministic—projections over the coordinates x and y (denoted herein by dx and dy) that turn out to display irregular shapes as found in a variety of geophysical applications and beyond (see, e.g., Puente, 2004).

In order to clarify the notions, Fig. 1 shows an example of a fractal wire passing through the three points $\{(0, 0), (0.50, -0.35), (1, -0.20)\}$ as generated by 10^6 iterations of the two maps

$$w_1\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.50 & 0 \\ -0.51 & -0.80 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{4}$$

and

$$w_2\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.50 & 0 \\ 0.03 & -0.60 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0.50 \\ -0.35 \end{pmatrix} \tag{5}$$

As may be readily verified, the two maps w_1 and w_2 satisfy the contractile Eqs. (2) and (3), operate in x over the intervals [0, 0.50] and [0.50, 1], respectively, and have vertical scaling parameters $d_1 = -0.8$ and $d_2 = -0.6$.

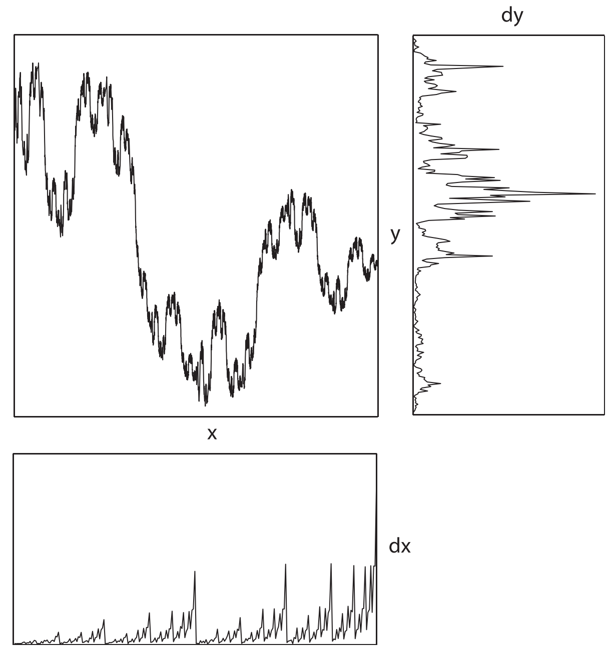

Fig. 1. The FM approach: from a multifractal dx to a projection dy via a fractal interpolating function, a wire from x to y.

In addition to the graph G, Fig. 1 also displays the projections (histograms) dx and dy, induced while carrying the previously mentioned chaos game according to a biased 30–70% proportion on w_1 and w_2.

As the x-coordinate in the maps is not affected by values of y (as implied by the zero entry in Eq. (1)), dx ends up being a simple deterministic (binomial) multifractal that, as it is related to a deterministic multiplicative cascade, exhibits noticeable repetition. In turn, dy happens to be the derived measure of dx via the fractal wire f and is computed, for any given value of y, by adding the corresponding "events" dx that satisfy $f(x) = y$. As can be seen, the geometrical FM construction generates a "random-looking" set dy that resembles a rainfall time series (e.g., Puente, 2004; Obregón et al., 2002b) and such is the basis for using such an approach, with suitable parameter values, to attempt to model hydrologic (geophysical) information.

Besides its clear geometric appeal, it happens that the FM approach may also be given a physical interpretation (Cortis et al., submitted for publication). For instance, as certain multifractals can be used to characterize energy distributions in turbulent atmospheric flows, the outputs dy may be interpreted as "reflections" (passive tracers) of turbulence or as non-trivial (fractal) integrations of rather spiky multifractals that reflect the phenomenology of random cascades. As the derived measures, for suitable sets of parameters, do share the spectrum of singularities of so-called "universal multifractals" (Tessier et al., 1993), they may also be thought of as specific realizations of random cascades, which have the advantage of being fully characterized, in their entirety, by a small set of parameters.

2.2. An extension with overlaps

The geometric procedure illustrated in Section 2.1 may be generalized so that the attractor G is no longer a function from x to y, but a "cloud" of points. This is accomplished by iterating N affine maps, as in Eq. (1), but replacing the contractive initial conditions by