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Special multiserial algebras are quotients of symmetric special multiserial algebras ☆

Edward L. Green [a], Sibylle Schroll [b,*]

[a] Department of Mathematics, Virginia Tech, Blacksburg, VA 24061, USA
[b] Department of Mathematics, University of Leicester, University Road, Leicester LE1 7RH, United Kingdom

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ABSTRACT

In this paper we give a new definition of symmetric special multiserial algebras in terms of *defining cycles*. As a consequence, we show that every special multiserial algebra is a quotient of a symmetric special multiserial algebra.

1. Introduction

A major breakthrough in representation theory of finite dimensional algebras is the classification of algebras in terms of their representation type. This is either finite, tame or wild [2]. Algebras of finite representation type have only finitely many isomorphism

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* Corresponding author.
E-mail addresses: green@math.vt.edu (E.L. Green), schroll@le.ac.uk (S. Schroll).

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classes of indecomposable modules, the infinitely many indecomposable modules of a tame algebra can be parametrized by one-parameter families whereas the representation theory of a wild algebra contains that of the free algebra in two generators and so in some sense contains that of any finite dimensional algebra. Thus no hope of a parametrization of the isomorphism classes of the indecomposable modules can exist.

For this reason, algebras of finite and tame representation type have been the focus of much of the representation theory of finite dimensional algebras. An important family of tame algebras are special biserial algebras defined in [12]. This class contains many of the tame group algebras of finite groups and tame subalgebras of group algebras of finite groups [3], gentle algebras, string algebras and symmetric special biserial algebras [14], also known as Brauer graph algebras [10,11], algebras of quasi-quaternion type [8] and the intensely studied Jacobian algebras of surface triangulations with marked points in the boundary arising in cluster theory [1].

The strength of the well-studied representation theory of special biserial algebras, derives from the underlying string combinatorics. Namely, by [4,9,14] every indecomposable non-projective module over a special biserial algebra is a string or band module. Not only does this give rise to a formidable tool for calculations and proofs but it also shows that special biserial algebras are of tame representation type.

Special multiserial algebras, defined in [13], are in general of wild representation type and as a consequence their indecomposable modules cannot be classified in a similar way. It is therefore remarkable that many of the results that are known to hold for special biserial algebras still hold for special multiserial algebras. For example, a very surprising fact about the indecomposable modules of these wild algebras was shown in [6]. Namely, the indecomposable modules over a special multiserial algebra are multiserial, that is, their radical is either 0 or a sum of uniserial submodules. Thus generalizing the analogous result for special biserial algebras [12]. However, given the absence of string combinatorics in the multiserial case, the proof is built on an entirely different strategy. The same holds true for the result and proofs in this paper.

We start by giving the definition of an algebra *defined by cycles*. This definition is built on the notion of a *defining pair*. We show that such an algebra is symmetric special multiserial and that conversely, every symmetric special multiserial algebra is an algebra defined by cycles. Note that in this context a symmetric algebra is an algebra over a field endowed with a symmetric linear form with no non-zero left ideal in its kernel. Symmetric algebras play an important role in representation theory and many examples of well-known algebras are symmetric such as group algebras of finite groups or Hecke algebras.

Given the new definitions of defining pairs and algebras defined by cycles we show that we can construct a defining pair for every special multiserial algebra A and that A is a quotient of the corresponding algebra defined by cycles. Thus we prove that every special multiserial algebra is a quotient of a symmetric special multiserial algebra. This result is an analogue of the corresponding result for special biserial algebras [14]. Moreover,