Journal of Functional Analysis 268 (2015) 3534–3565

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Weak expansiveness for actions of sofic groups

Nhan-Phu Chung [a,*], Guohua Zhang [b]

[a] *Department of Mathematics, University of Sciences, Vietnam National University at Ho Chi Minh City, Viet Nam*
[b] *School of Mathematical Sciences and LMNS, Fudan University and Shanghai Center for Mathematical Sciences, Shanghai 200433, China*

ARTICLE INFO

Article history:
Received 6 October 2014
Accepted 19 December 2014
Available online 6 January 2015
Communicated by S. Vaes

MSC:
primary 37B05
secondary 37C85, 54H15

Keywords:
Actions of sofic groups
Expansive
Asymptotically h-expansive
Measures with maximal entropy

ABSTRACT

In this paper, we shall introduce h-expansiveness and asymptotical h-expansiveness for actions of sofic groups. By definition, each h-expansive action of a sofic group is asymptotically h-expansive. We show that each expansive action of a sofic group is h-expansive, and, for any given asymptotically h-expansive action of a sofic group, the entropy function (with respect to measures) is upper semi-continuous and hence the system admits a measure with maximal entropy.

Observe that asymptotically h-expansive property was first introduced and studied by Misiurewicz for $\mathbb{Z}$-actions using the language of tail entropy. And thus in the remaining part of the paper, we shall compare our definitions of weak expansiveness for actions of sofic groups with the definitions given in the same spirit of Misiurewicz's ideas when the group is amenable. It turns out that these two definitions are equivalent in this setting.

* Corresponding author.
E-mail addresses: cnphu@hcmus.edu.vn (N.-P. Chung), chiaths.zhang@gmail.com (G. Zhang).

http://dx.doi.org/10.1016/j.jfa.2014.12.013

1. Introduction

Dynamical system theory is the study of qualitative properties of group actions on spaces with certain structures. In this paper, by a *topological dynamical system* we mean a continuous action of a countable discrete sofic group on a compact metric space. Sofic groups were defined implicitly by Gromov in [26] and explicitly by Weiss in [48]. They include all amenable groups and residually finite groups.

Recently, Lewis Bowen introduced a notion of entropy for measure-preserving actions of a countable discrete sofic group admitting a generating measurable partition with finite entropy [7,9]. The main idea here is to replace the important Følner sequence of a countable discrete amenable group with a sofic approximation for a countable discrete sofic group. Very soon after [7], in the spirit of L. Bowen's sofic measure-theoretic entropy, Kerr and Li developed an operator-algebraic approach to sofic entropy [32,34] which applies not only to continuous actions of countable discrete sofic groups on compact metric spaces but also to all measure-preserving actions of countable discrete sofic groups on standard probability measure spaces. From then on, there are many other papers, presenting different but equivalent definitions of sofic entropy [31,50], extending sofic entropy to sofic pressure [14] and to sofic mean dimension [35], and discussing combinatorial independence for actions of sofic groups [33].

Let X be a compact metric space. Any homeomorphism $T : X \to X$ generates naturally a topological dynamical system by considering the group $\{T^n : n \in \mathbb{Z}\}$. Even in the case the given map $T : X \to X$ is just a continuous map (may be non-invertible), we still call it a topological dynamical system by considering the semi-group $\{T^n : n \in \mathbb{Z}_+\}$. A self-homeomorphism of a compact metric space is *expansive* if, for each pair of distinct points, some iterate of the homeomorphism separates them by a definite amount. Expansiveness is in fact a multifaceted dynamical condition which plays a very important role in the exploitation of hyperbolicity in smooth dynamical systems [39]. In the setting of considering a continuous mapping over a compact metric space, two classes of weak expansiveness, the h-expansiveness and asymptotical h-expansiveness, were introduced by Rufus Bowen [6] and Misiurewicz [40], respectively. By definition, each h-expansive system is asymptotically h-expansive. Both of h-expansiveness and asymptotical h-expansiveness turn out to be important in the study of smooth dynamical systems [12,18,21,22,37].

It is direct to define expansiveness for actions of groups. That is, let G be a discrete group acting on a compact metric space X (with the metric ρ), then we say that (X, G) *is expansive* if there is $\delta > 0$ such that for any two different points x_1 and x_2 in X there exists $g \in G$ with $\rho(gx_1, gx_2) > \delta$. Such a δ is called an *expansive constant*. Symbolic systems are standard examples for expansive actions. This obvious extension of the notion of expansiveness has been investigated extensively in algebraic actions for $\mathbb{Z}^d$ [24,38,44,45] and for more general groups [8,15,17]; and a general framework of dynamics of $d \in \mathbb{N}$ commuting homeomorphisms over a compact metric space, in terms of expansive behavior along lower dimensional subspaces of $\mathbb{R}^d$, was first proposed by Boyle and Lind [11].