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Large-time behavior of solutions to the equations of a viscous heat-conducting flow with shear viscosity in unbounded domains

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ABSTRACT

We consider the initial and initial–boundary value problems for a viscous heat-conducting flow with shear viscosity in unbounded domains with general large initial data. We prove that the temperature is bounded from below and above uniformly in time and space and that the global solution is asymptotically stable as the time tends to infinity. Our approach relies upon the energy-estimate method.

1. Introduction

1.1. The Eulerian description

We study the initial and initial–boundary value problems for the equations describing a viscous compressible heat-conducting flow between two horizontal plates:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + \left(\rho u^2 + P\right)_x = (\mu u_x)_x, \\ (\rho \boldsymbol{w})_t + (\rho u \boldsymbol{w})_x = (\lambda \boldsymbol{w}_x)_x \,, \\ (\rho E)_t + (\rho u E + P u)_x = (\kappa \theta_x + \mu u u_x + \lambda \boldsymbol{w} \cdot \boldsymbol{w}_x)_x \,, \end{cases} \tag{1.1}$$

where $t > 0$ and $x \in \mathbb{R}$ are the time variable and the spatial variable, respectively, and the primary dependent variables are the fluid density ρ, the longitudinal velocity $u \in \mathbb{R}$, the transverse velocity $\boldsymbol{w} \in \mathbb{R}^2$, and the temperature θ. The specific total energy is $E = e + \frac{1}{2}u^2 + \frac{1}{2}|\boldsymbol{w}|^2$ with e being the specific internal energy.

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The pressure P and the specific initial energy e are related with the density and temperature by means of constitutive relations

$$(P, e) = (P(\rho, \theta), e(\rho, \theta)).$$

The viscosity coefficient μ, the shear viscosity coefficient λ and the heat conductivity κ may depend on both ρ and θ generally. One can easily derive (1.1) from the three-dimensional compressible Navier–Stokes equations for a Newtonian fluid (see Wang [15]).

In this article, we focus on the ideal polytropic gases, which are identified by the constitutive relations

$$P = R\rho\theta, \quad e = c_v\theta, \tag{1.2}$$

where R is the gas constant and c_v is the specific heat at constant volume. And we assume that μ, λ, κ and c_v are positive constants.

The system (1.1) is supplemented with the initial data

$$(\rho(0,x), u(0,x), \boldsymbol{w}(0,x), \theta(0,x)) = (\rho_0(x), u_0(x), \boldsymbol{w}_0(x), \theta_0(x)) \quad \text{for } x \in \mathbb{R}, \tag{1.3}$$

and one type of the following far-field and boundary conditions:

1) far-field conditions for $\Omega = \mathbb{R}$,

$$\lim_{x\to\pm\infty} (\rho_0(x), u_0(x), \boldsymbol{w}_0(x), \theta_0(x)) = (1, 0, 0, 1); \tag{1.4}$$

2) boundary and far-field conditions for $\Omega = (0, \infty)$,

$$(u(t,0), \boldsymbol{w}(t,0), \theta(t,0)) = (0,0,1), \quad \lim_{x\to\infty} (\rho_0(x), u_0(x), \boldsymbol{w}_0(x), \theta_0(x)) = (1, 0, 0, 1); \tag{1.5}$$

3) boundary and far-field conditions for $\Omega = (0, \infty)$,

$$(u(t,0), \boldsymbol{w}(t,0), \theta_x(t,0)) = (0,0,0), \quad \lim_{x\to\infty} (\rho_0(x), u_0(x), \boldsymbol{w}_0(x), \theta_0(x)) = (1, 0, 0, 1). \tag{1.6}$$

Let us mention some results on the system (1.1) without any restrictions on the smallness of the initial data. Wang [15] investigated the existence, uniqueness and regularity of global solutions to system (1.1) in bounded domains under certain assumptions on the constitutive relations. Qin and Hu [14] improved and extended the results in [15] for more general constitutive relations. We note here that the case of ideal polytropic gases (1.2) is not included in the class of fluids established in [15] and [14].

In this paper we shall show the global existence and uniqueness of solutions to the Cauchy problem (1.1)–(1.4) and the initial–boundary value problems (1.1)–(1.3), (1.5) and (1.1)–(1.3), (1.6) for general initial data. To investigate the global solvability of the system (1.1), some difficulties arise for the case of unbounded domains, where the imbedding $L^2 \hookrightarrow L^1$ is not valid any longer. We will introduce the Lagrangian variable and transform the problems (1.1)–(1.4), (1.1)–(1.3), (1.5), and (1.1)–(1.3), (1.6) in the Eulerian coordinates into corresponding problems in the Lagrangian coordinates. The local existence and uniqueness of solutions can be shown by using the Banach theorem and the contractivity of the operator defined by the linearization of the problems on a small time-interval. The global solvability is proved by using the continuation argument to extend the local solutions globally in time based on the global a priori estimates of solutions.