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A novel single-period inventory problem with uncertain random demand and its application

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ABSTRACT

In many real inventory situations of short life-cycle products the decision maker often has to provide a subjective estimate of new demand distribution due to the lack of historical data. Thus, both randomness and uncertainty simultaneously appear in a single-period inventory (newsboy) problem. In this paper, we develop both single-item and multi-item single-period inventory models when market demands are assumed to be uncertain random variables. The objective of this study is to provide theoretical analysis of the models that attains optimality when demand information availability in subjective judgments leading to uncertainty along with random variation. The uncertain random models are transferred to equivalent deterministic forms by considering expected profit and providing more information of chance distributions. Finally, the numerical examples of ordering pharmaceutical reference standard materials are presented to illustrate the models.

1. Introduction

Decision making in inventory management is always accompanied by uncertainties especially when dealing with demand related to new innovative products, seasonal products, sport goods or fashion goods etc. The characteristic of short selling season and long lead times for such products provide only one order opportunity and hence inventory problem can be solved properly as single-period decision-making setting. Thus, single-period inventory problem (popularly known as the Newsboy problem) is often formulated as a classical production/inventory management problem, which was primarily studied by Hadley and Whitin [4].

Most of the extensions of single-period inventory problem have been made in the probabilistic framework, in which the uncertainty of demand is characterized by the random demands. After Hadley and Whitin's pioneering work, both single-item case and multi-item case with constraints have been widely studied in the area of production/inventory management. By assuming random demand, Nahmias and Schmidt [17] considered a multi-item single-period inventory problem subject to linear and deterministic constraints on space or budget. Moon and Silver [16] continued considering the multi-item issue subject to not only a budget constraint on the total value of the replenishment quantities but also fixed costs for non-zero replenishment. Vairaktarakis [28] focused on how to describe uncertainty and discussed the multi-item problem with budget constraint by using interval and discrete demand scenarios. Olzer et al. [18] utilized value-at-risk as the risk measure and investigated the

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multi-product newsvendor problem under a value-at-risk constraint. As a fundamental management tool, single-period inventory problem has actually always attracted more attentions. Some recent developments are devoted to the extensions of the basic model. For example, Kamburowski [6] provided new theoretical foundations for analyzing the case under incomplete information about probability distribution of random demand, Rossi et al. [24] introduced a novel strategy to address the issue of demand estimation by combining confidence interval analysis and inventory optimization, Wu et al. [31] studied a risk-averse situation with quantity competition and price competition based on conditional value-at-risk criterion and Sayın et al. [25] considered both random demand and random supply and provided the optimal ordering policy and optimal portfolio at the same time. For the case with partial information, several authors such as Qiu and Shang [23], Turgay et al. [27] and Wang et al. [29] also apply robust optimization to handle the distribution uncertainty of probability parameters in the newsvendor or inventory problems.

However, randomness is not the unique uncertainty to deal with the real world inventory problems. Sometimes the probability distributions of the demands for products are difficult to acquire due to lack of information and historical data. In such a case, the demands are approximately specified based on the experience and subjective judgments of decision makers/experts. The decision maker has to provide the belief degree to describe the market demand. The uncertainty theory, founded by Liu [8] is a feasible tool that deals with such uncertainties or belief degrees. Based on this framework, Liu [9] further proposed uncertain programming for solving optimization problems involving uncertain variables. In this connection one may refer to the recent works on different applications of uncertainty theory such as risk analysis [10], shortest path problem [3], portfolio optimization problem [5,15], insurance risk model [11], facility location-allocation [30], new product development [32], p-hub center location [19] and so on. Particularly, Qin and Kar [20] first extended the single-period inventory model by considering market demand as an uncertain variable. On this basis, Ding [1] extended Qin and Kar's work to multi-product case by adding chance constraint.

No matter random models (Hadley and Whitin [4]) or uncertain models (Qin and Kar [20]) only handles single-fold uncertainty in the single-period inventory problem. However, in many real situations, randomness and uncertainty often exist simultaneously in a complex system. To describe such a system, Liu [12] first proposed the concept of uncertain random variable and a new chance measure to measure the possibility of a hybrid event. Correspondingly, Liu [12] also presented the operational law of uncertain random variable and introduced the related concepts such as chance distribution, expected value, variance and so on. As a general theoretical framework to model the practical problems with unknown parameters, uncertain random programming was introduced by Liu [13] and then extended to uncertain random multi-objective programming [33], uncertain random multilevel programming [7] and uncertain random goal programming [21]. As applications, Sheng and Gao [26] considered arc capacities of a network as random variables or uncertain variables and then derived the chance distribution of the maximum flow. Qin [22] presented a mean-variance model for portfolio optimization problem with mixture of random and uncertain returns. Liu and Ralescu [14] defined a risk index to quantify the risk of a system with uncertain random parameters, and Ding [2] introduced single-product newsboy problem with the demand consisting of random variable and uncertain variable.

In some real-life inventory problems, demand parameter is uncertain and diversity of events may cause the lack of data. So we have to invite some experts to evaluate their degree of belief that each event will occur. For instance, the demands for different pharmaceutical reference standard materials are determined by the decision makers through integrating their subjective judgments and the historical demands for similar goods. Motivated by this point, we assume that the demands are described by uncertain random variables and develop novel single-period inventory models for such decision environments. The main contribution of this paper is to provide a more general framework for single-period inventory problem by considering single-item and multiple items with a budget constraint, respectively. The proposed models capture both uncertain and random behavior of the demands and cover not only the random case but also the single-fold uncertain case. They are more suitable for determining an unambiguous optimal order quantity when the demands are estimated by combining experts' point of view and some small amount of historical data.

The rest of the paper is organized as follows. In Section 2, we review the necessary preliminaries related to uncertain measure, uncertain variable and uncertain random variable. In Section 3, an uncertain random single-period inventory model is constructed for single item and some analytical findings are provided related to the optimal order quantity. Section 4 develops the multi-item situation by introducing a budget constraint and the equivalent deterministic forms are given for simple and mixed uncertain random demands. The numerical examples of ordering pharmaceutical reference standard materials are presented and analyzed in Section 5. Finally, we conclude the paper in Section 6.

2. Preliminaries

In this section, we review some preliminaries about uncertain measure, uncertain variable and uncertain random variable. The first concept is uncertain measure, which was proposed by Liu [8] to indicate the possibility that a possible event happens. Let Γ be a nonempty set and $\mathcal{L}$ a σ-algebra on it. A set function $\mathcal{M}: \mathcal{L} \to [0, 1]$ is called an uncertain measure by Liu [8] if it satisfies: (1) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ; (2) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event $\Lambda \in \mathcal{L}$; (3) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have $\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$. The triple $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertain space. If $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ are uncertainty spaces for $k = 1, 2, \ldots$, then Liu [9] defined the product uncertain measure $\mathcal{M}$ as an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.