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Global synchronization of uncertain chaotic systems via discrete-time sliding mode control

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ABSTRACT

This paper presents a discrete-time sliding mode control scheme for a class of master–slave (or drive-response) chaotic synchronization systems. The proposed scheme guarantees the stability of closed-loop system and achieves the global synchronization between the master and slave systems. The structure of slave system is simple and needs not be identical to the master system. Moreover, the selection of switching surface and the existence of sliding mode have been addressed. Numerical simulations are given to validate the proposed synchronization approach.

1. Introduction

Chaotic behavior is an interesting phenomenon appearing nonlinear systems and has been received more and more attentions in the last decades. A chaotic system is a highly complex dynamic nonlinear system. The prominent characteristic of a chaotic system is its extreme sensitivity to initial conditions and the system's parameters, and this makes the problem of chaotic synchronization much more important. In the last few years, chaotic synchronization has applied in vast area of physics and engineering systems such as in chemical reactions, power converter, biological systems, information processing, especially in secure communication [1–3]. Many methods have been developed to realize the problem of the synchronization of chaotic systems including state feedback method [1,2,4–10], the observer method [3,11–15] and output feedback method [16]. However, these methods are developed in continuous-time system. To the best of the author's knowledge, the problem of synchronizing uncertain chaotic systems in discrete-time domain has not been fully investigated and is still open in the literature. This has motivated our research.

On the other hand, using computers or DSP chips to implement the controller has become more and more important nowadays. Therefore, research in discrete-time control has been intensified in recent years, and it is quite natural to extend the technique of continuous control to discrete-time systems. Sliding mode control (SMC) is a nonlinear control approach. The continuous-time SMC is known as a robust method and has attractive features such as fast response, good transient performance, insensitiveness to the matching parameter uncertainties and external disturbances [17,18]. Over the past few years, it has been widely applied to many practical control systems. Several design methods of discrete-time SMC have been proposed in the literature [19–23].

In this paper, a discrete-time SMC scheme is developed to control the synchronization of a class of uncertain chaotic systems in the master–slave (or drive-response) framework. The proposed scheme has the following attractive features: (1) the control design is rather straightforward and ensures the synchronization of the master–slave chaotic systems, (2) the structure of slave system is simple and needs not be identical to the master system, (3) the discrete-time SMC needs not a switching type of control law. Chattering phenomenon and reaching phase are eliminated, (4) the control strategy can be easily

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applied to other dimensional chaotic synchronization problems. The organization of this paper is as follows. Section 2 briefly states the master–slave chaotic synchronization systems. Section 3 provides the proposed discrete-time SMC scheme. Section 4 presents results from numerical simulations. Finally, a conclusion is provided in Section 5.

2. System description and problem formulation

Consider a class of chaotic systems described by

$$\dot{x}(t) = Ax(t) + Bg(x,t), \tag{1}$$

where $x(t) \in R^n$ is the state vector, $g(\cdot) \in R^r$ represents the nonlinear function vector that may include unknown parameter perturbations and external disturbances. The constant matrices A and B are known constant matrices of appropriate dimensions.

Remark 1. It should be noted that chaotic system (1) is quite popular and has been used in many studies in the literature, such as Lorenz system [4,6,24], Chen chaotic dynamical system [5], Duffing–Holmes system [7], Chua's circuit [8], and Chen–Lee system [25].

The discrete-time representation of system (1) with sample and hold process is given by

$$x(k+1) = \Phi x(k) + \Gamma g(k), \tag{2}$$

where the sampling time is T, $\Phi = e^{AT}$ and $\Gamma = \int_0^T e^{A\tau} d\tau B$ [22]. The main objective of this paper is to find a discrete-time sliding mode controller to synchronize the discrete-time chaotic system (2) in the master–slave (or drive-response) framework. To facilitate further development, we make the following assumptions:

Assumption 1. The pair (Φ, Γ) is controllable.

Assumption 2. The sampling interval T is assumed to be sufficiently small such that the nonlinear function $g(k)$ does not vary too much between consecutive sampling instances. Also, there exists a positive constants ρ such that

$$||g(k)|| \leqslant \rho < \infty. \tag{3}$$

In this paper, for the general class of discrete-time chaotic system (2), the master and slave systems are respectively defined as follows:

$$x_m(k+1) = \Phi x_m(k) + \Gamma g(k), \tag{4}$$

and

$$x_s(k+1) = \Phi x_s(k) + \Gamma u(k), \tag{5}$$

where $x_m(k) \in R^n$ and $x_s(k) \in R^n$ are the master system's state vector and slave system's state vector, respectively, $u(k) \in R^m$ is the control input vector, and $||g(k)|| \leqslant \rho < \infty$.

Let us define the synchronization error $e(k)$ vector as

$$e(k) = x_m(k) - x_s(k). \tag{6}$$

The dynamics of synchronization error between the master and slave systems given in (4) and (5) can be described by

$$e(k+1) = \Phi e(k) + \Gamma g(k) - \Gamma u(k). \tag{7}$$

It is clear that the synchronization problem is replaced by the equivalent problem of stabilizing the synchronization error system (7) using a suitable choice of the control law $u(k)$. In the sequel, using the proposed discrete-time SMC scheme, the asymptotical stability of synchronization error system (7) can be achieved in the sense that $||e(k)|| \to 0$ as $k \to \infty$.

Remark 2. The sliding mode characteristics of discrete-time SMC systems are different from those of continuous-time SMC systems. It is noted that the motion of a discrete-time SMC system can approach the switching surface but cannot stay on it in practice. Thus, only the quasi-sliding mode is ensured [19–21].

3. Switching surface and discrete-time sliding mode controller design

In this paper, the switching function is defined as follows:

$$s(k) = Ge(k) - G\exp(-\beta k)e(0) - \varepsilon(k), \quad \beta > 0, \tag{8a}$$

$$\varepsilon(k) = \varepsilon(k-1) + G(\Phi + \Gamma K)e(k-1), \quad \varepsilon(0) = 0, \tag{8b}$$